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NETWORK ANALYSIS AND TRANSMISSION LINES
COURSE FILE

DEPARTMENT OFELECTRONICS & COMMUNICATIONENGINEERING
(2022-2023)

Faculty In-Charge | **HOD-ECE**
CH.RAJASHEKAR | **Dr.R.Prabhakar**

COURSE FILE

SUBJECT: NETWORK ANALYSIS AND TRANSMISSION LINES

ACADEMIC YEAR: 2022-2023.

REGULATION:R18

NAMEOFTHEFACULTY:C H . R A J A S H E K A R

DEPARTMENT:ECE

YEAR&SECTION:II ECE A,B,C

SUBJECTCODE:

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1.PEO'S,PO'S,PSO'S

PROGRAMEDUCATIONALOBJECTIVES:
PEO1: To excel in different fields of electronics and communication as well as in multidisciplinary areas. This can lead to a new era in developing a good electronic product.
PEO2: To increase the ability and confidence among the students to solve any problem in their profession by applying mathematical, scientific and engineering methods in a better and efficient way.
PEO3: To provide a good academic environment to the students which can lead to excellence, and stress upon the importance of teamwork and good leadership qualities, written ethical codes and guide lines for lifelong learning needed for a successful professional career.
PEO4: To provide student with a solid foundation to students in all areas like mathematics, science and engineering fundamentals required to solve engineering problems, and also to pursue higher studies.
PEO5: To expose the student to the state of art technology so that the student would be in a position to take up any assignment after his graduation.

PROGRAMOUTCOMES:-
Engineeringknowledge:Applytheknowledgeofmathematics,science,engineeringfundamentals,and anengineering specializationto thesolutionofcomplexengineering problems.
Problemanalysis:Identify,formulate,reviewresearchliterature,andanalyzecomplexengineeringproblems reaching substantiated conclusions using first principles of mathematics, natural sciences,and engineeringsciences.
Design/developmentofsolutions: Designsolutionsforcomplexengineeringproblemsanddesignsystem components or processes that meet the specified needs with appropriate consideration forthepublichealthand safety,and thecultural,societal,and environmental considerations.
Conduct investigations of complex problems: Use research-based knowledge and researchmethodsincludingdesignofexperiments,analysisandinterpretationofdata,andsynthesisoftheinformationtoprovidevalidconclusions.
Modern tool usage: Create, select, and apply appropriate techniques, resources, and modernengineeringandITtoolsincludingpredictionandmodelingtocomplexengineeringactivitieswithanunderstanding ofthelimitations.
Theengineerandsociety:Applyreasoninginformedbythecontextualknowledgetoassesssocietal,health, safety, legal and cultural issues and the consequent responsibilities relevant to theprofessional engineeringpractice.
Environment and sustainability: Understand the impact of the professional engineering solutionsinsocietalandenvironmentalcontexts,anddemonstratetheknowledgeof,andneedforsustainabledevelopment.
Ethics: Apply ethical principles and commit to professional ethics and responsibilities and norms oftheengineeringpractice.
Individualandteam work:Functioneffectivelyasanindividual,andasamemberorleaderindiverseteams,andin multidisciplinarysettings.
Communication: Communicate effectively on complex engineering activities with the engineeringcommunityandwithsocietyatlarge,suchas,beingableto comprehendandwriteeffectivereportsanddesigndocumentation,makeeffectivepresentations,andgiveandreceiveclearinstructions.
Project management and finance: Demonstrate knowledge and understanding of the engineeringandmanagementprinciplesandapplytheseto one'sownwork,asamemberandleaderinateam,tomanageprojectsandin multidisciplinaryenvironments.

Life-longlearning:Recognizetheneedfor,andhavethepreparationandabilityto engageinindependentand life-long learning inthebroadestcontextoftechnological change.

PROGRAMSPECIFIC OUTCOMES:

PSO1: The ability to absorb and apply fundamental knowledge of core Electronics and Communication Engineering subjects in the analysis, design, and development of various types of integrated electronic systems as well as to interpret and synthesize the experimental data leading to valid conclusions.

PSO2: Competence in using electronic modern IT tools (both software and hardware) for the design and analysis of complex electronic systems in furtherance to research activities.

PSO3: Excellent adaptability to changing work environment, good interpersonal skills as a leader in a team in appreciation of professional ethics and societal responsibilities.

EC302PC: NETWORK ANALYSIS AND TRANSMISSION LINES

B.Tech. II Year I Sem.

L	T	P	C
3	0	0	3

Pre-Requisites: Nil

Course Objectives:

- To understand the basic concepts on RLC circuits.
- To know the behavior of the steady states and transients states in RLC circuits.
- To understand the two port network parameters.
- To study the propagation, reflection and transmission of plane waves in bounded andunbounded media.

Course Outcomes: Upon successful completion of the course, students will be able to:

- Gain the knowledge on basic RLC circuits behavior.
- Analyze the Steady state and transient analysis of RLC Circuits.
- Know the characteristics of two port network parameters.
- Analyze the transmission line parameters and configurations.

UNIT - I

Network Topology, Basic cutset and tie set matrices for planar networks, Magnetic Circuits, Self and Mutual inductances, dot convention, impedance, reactance concept, Impedance transformation and coupled circuits, co-efficient of coupling, equivalent T for Magnetically coupled circuits, Ideal Transformer.

UNIT - II

Transient and Steady state analysis of RC, RL and RLC Circuits, Sinusoidal, Step and Square responses. RC Circuits as integrator and differentiators. 2^{nd} order series and parallel RLC Circuits, Rootlocus, damping factor, over damped, under damped, critically damped cases, quality factor and bandwidth for series and parallel resonance, resonance curves.

UNIT - III

Two port network parameters, Z, Y, ABCD, h and g parameters, Characteristic impedance, Image transfer constant, image and iterative impedance, network function, driving point and transfer functions

– using transformed (S) variables, Poles and Zeros. Standard T, π, L Sections, Characteristic impedance, image transfer constants, Design of Attenuators, impedance matching network.

UNIT – IV

Transmission Lines - I: Types, Parameters, Transmission Line Equations, Primary & Secondary Constants, Equivalent Circuit, Characteristic Impedance, Propagation Constant, Phase and Group Velocities, Infinite Line Concepts, Lossless / Low Loss Characterization, Types of Distortion, Condition for Distortion less line, Minimum Attenuation, Loading - Types of Loading.

UNIT – V

Transmission Lines – II: Input Impedance Relations, SC and OC Lines, Reflection Coefficient, VSWR. $\lambda/4$, $\lambda/2$, $\lambda/8$ Lines – Impedance Transformations, Smith Chart – Configuration and Applications, Single Stub Matching.

TEXT BOOKS:

1. Network Analysis – Van Valkenburg, 3rd Ed., Pearson, 2016.
2. Networks, Lines and Fields - JD Ryder, PHI, 2nd Edition, 1999.

REFERENCE BOOKS:

1. Electric Circuits – J. Edminister and M. Nahvi – Schaum's Outlines, Mc Graw Hills Education,1999.
2. Engineering Circuit Analysis – William Hayt and Jack E Kemmerly, MGH, 8th Edition, 1993.
3. Electromagnetics with Applications – JD. Kraus, 5th Ed., TMH
4. Transmission Lines and Networks – Umesh Sinha, Satya Prakashan, 2001, (Tech. India Publications), New Delhi.

3.ClassTimeTable&IndividualTimeTable

Class: II/IV B.Tech – I Semester LECTURE HALL – B1 G04 Branch: ECE-A

ELECTRONICS AND COMMUNICATION ENGINEERING DEPARTMENT

Day/ Time	9:15 am to 10:15 am	10:15 am to 11:15 am	11:15 am to 12:15 pm	12:15 pm to 1:15 pm	1:15pm to 2:00 pm	2:00 pm to 3:00 pm	3:00 pm to 4:00 pm
Monday	EDC	DSD	NATL	PTSP	LUNCH	DSD LAB/EDC LAB	
Tuesday	NATL	PTSP	DSD	SS		EDC	LIBRARY
Wednesday	DSD	PTSP	EDC	NATL		SS	SEMINAR
Thursday	SS	EDC	EDC LAB/DSD LAB			PTSP	TUTORIAL
Friday	NATL	SS	PTSP	DSD		COI	SPORTS
Saturday	SS	NATL	DSD	EDC		BS LAB	

Class: II/IV B.Tech – I Semester
LECTURE HALL – B1 G 07 Branch: ECE-B

ELECTRONICS AND COMMUNICATION ENGINEERING DEPARTMENT

Day/ Time	9:15 am to 10:15 am	10:15 am to 11:15 am	11:15 am to 12:15 pm	12:15 pm to 1:15 pm	1:15pm to 2:00 pm	2:00 pm to 3:00 pm	3:00 pm to 4:00 pm
Monday	SS	NATL	EDC	DSD	LUNCH	PTSP	SPORTS
Tuesday	DSD	EDC	DSD LAB/EDC LAB			SS	LIBRARY
Wednesday	PTSP	SS	DSD	EDC		NATL	SEMINAR
Thursday	NATL	DSD	SS	PTSP		BS LAB	
Friday	DSD	PTSP	NATL	SS		EDC	TUTORIAL
Saturday	EDC	NATL	PTSP	COI		EDC LAB/DSD LAB	

Individual TimeTable:

	9.15-10.15	10.15-11.15	11.15-12.15	12.15-1.15	1.15-2.00	2.00-3.00	3.00-4.00
MON		NATL(B)	NATL(A)	(B)	**LUNCH**		
TUES			NATL(A)				
WED				NATL(A)		NATL(B)	
THUR	NATL(B)						
FRI	NATL(A)		NATL(B)				
SAT		NATL(B)	NATL(A)				

4.StudentsRollList ECE-A

SNo	H.T.NO	NAME OF THE STUDENT	SNo	H.T.NO	NAME OF THE STUDENT
1	21S11A0401	ABHIRAM TALLA	27	21S11A0427	RAHITH KUMAR KANDLAGUNTA
2	21S11A0402	AKASH BASHETTY	28	21S11A0428	RAJESHWAR J
3	21S11A0403	AKSHAY KUMAR REDDY KUNCHANAGARI	29	21S11A0429	RANI ANANTHA
4	21S11A0404	ANJANEYULU KAMMARI	30	21S11A0430	REKHA MANGA
5	21S11A0405	ANKIT RAJ	31	21S11A0431	REVATHI MEESALA
6	21S11A0406	ASAD PASHA SHAIK	32	21S11A0432	RISHAB SAKALE
7	21S11A0407	ASHWINI CHETHIPATTI	33	21S11A0433	SAI KRISHNA REDDY B
8	21S11A0408	BHARATH K	34	21S11A0434	SAI RATNA VEMULA
9	21S11A0409	BHEESHMA SANDI	35	21S11A0435	SAI RITHIK SIBYALA
10	21S11A0410	CHAITHANYA ANUMANCHINENI	36	21S11A0436	SAI SRIYA PETTEM
11	21S11A0411	CHANTI BODA	37	21S11A0437	SAI VENKATA KRISHNA MRUDUL RAYANAPATI
12	21S11A0412	DARSHAN KUMBAM	38	21S11A0438	SHANKHABRATA ROY
13	21S11A0413	GANESH VANKUDOTH	39	21S11A0439	SHARATH CHANDRA REDDY YALLA
14	21S11A0414	GEETA RAGHUJI REDDY	40	21S11A0440	SHIVA SAI REDDY SHAGAM
15	21S11A0415	HARIKA SATTI	41	21S11A0441	SHIVA SHANKAR BADDULA
16	21S11A0416	HASINI BASHETTY	42	21S11A0442	SREENIPA NANDELLI
17	21S11A0417	JAGADEESH SANGHISHETTY	43	21S11A0443	SRIRAM REDDY ANANTHA
18	21S11A0418	JAYA PRAKASH REDDY PANYALA	44	21S11A0444	SWATHI KASHAPAKA
19	21S11A0419	JEEVANA GATLA	45	21S11A0445	SYED FAHAD
20	21S11A0420	KALYANI JULKAPELLI	46	21S11A0446	TUSHWANTH KARUTURI
21	21S11A0421	MANISHA MULA	47	21S11A0447	VAISHNAVI DEVA
22	21S11A0422	MEHAR NIKHIL MANNE	48	21S11A0448	VENKAT RAO THOKALA
23	21S11A0423	NANDINI MANNE	49	21S11A0449	VENKATA NAGA VARSHITHA POLISETTY
24	21S11A0424	NITISH REDDY KOTHAKAPU	50	21S11A0450	VIJAY KUMAR KASAM
25	21S11A0425	PAVAN KUMAR MALLAPPAGARI	51	21S11A0451	VINAY SANGEM
26	21S11A0426	PRAKASHAM VADAPARTHI	52	21S11A0452	VISHNU VANGARI

SNo	H.T.NO	NAME OF THE STUDENT	SNo	H.T.NO	NAME OF THE STUDENT
1	21S11A0453	AJAY KUMAR REDDY VITTA	27	21S11A0479	POONAM SAHU
2	21S11A0454	AKHILA BHUKYA	28	21S11A0480	PRAKASH KATLA
3	21S11A0455	AKSHAY GOUD DURGAM	29	21S11A0481	PREMKANTH KOMMINENI

4	21S11A0456	AKSHAY MIRUPALA	30	21S11A0482	RAJENDER VANKUDOTH
5	21S11A0457	ANJANEYULU B	31	21S11A0483	RAKESH KRISHNA JAKKA
6	21S11A0458	ARJUN VISLAVATH	32	21S11A0484	ROHITH REDDY PULAKANTI
7	21S11A0459	BHANU SAI NAGENDER PAPPALA	33	21S11A0485	SAI KUMAR REDDY MANDAPATI
8	21S11A0460	BHARGAVI MANDHUGULA	34	21S11A0486	SAI PRASAD K
9	21S11A0461	CHETHAN THEEGALA	35	21S11A0487	SAI PRASAD REDDY AKKENAPALLY
10	21S11A0462	DEVI PRIYANKA NARIKALAPA	36	21S11A0488	SAICHAND KARRA
11	21S11A0463	ESHWAR BOLLAPALLI	37	21S11A0489	SAINADH TEEGALA
12	21S11A0464	ESHWAR VENKATA SATYA SAI VITTANALA	38	21S11A0490	SAITEJA KODHATI
13	21S11A0465	GANGADHAR REDDY CHALLA	39	21S11A0491	SAKETHBABU VARAGANI
14	21S11A0466	JAI SINGH ROTHVAN	40	21S11A0492	SIDDARTHA YADAV THOTLA
15	21S11A0467	JEEVAMRUTHA AKARAPU	41	21S11A0493	SIVA KIRAN AKSHINTALA
16	21S11A0468	KARTHIK KUMAR C	42	21S11A0494	SPANDANA SEEDULA
17	21S11A0469	KRISHNA TOLUPUNURI	43	21S11A0495	SRIRAM SINGARAM
18	21S11A0470	MAHESH NOMULA	44	21S11A0496	SRIVANI GEDDADA
19	21S11A0471	MANI VEERA NAGENDRA DASARI	45	21S11A0497	SUDHEER KUMAR TOKALA
20	21S11A0472	MANOJ KUMAR VELISHALA	46	21S11A0498	TEJA SRI GURRALA
21	21S11A0473	NAGA RAJU RAVULA	47	21S11A0499	THANU SRI REDDY MALLE
22	21S11A0474	NAGARAJU ARUGONDA	48	21S11A04A0	VAISHNAVI CHEDDE
23	21S11A0475	NEETHU BOKKA	49	21S11A04A1	VAMSHI KRISHNA AMARAGONDA
24	21S11A0476	NIKHITHA GANGALA	50	21S11A04A2	VIGNESH VALAGIRI
25	21S11A0477	PAVAN KUMAR UPUTURI	51	21S11A04A3	SYED KALEEMULLAH HUSSAIN
26	21S11A0478	PAVAN YALKAPALLY	52	21S11A04A4	RICHA MIDDE

LESSON PLAN:

Session	Week	Unit	Topics	Course Learning Outcomes	Learning Reference
			Review of R, L,C	**Know** about electricalelements	T1,T2,R1

	1.	I.	Review of RC, RL, RLC circuits	**Analyse** RC, RL, RLC circuits	T1,T2,R1
			Network Topology, Terminology	**Understanding** Network	T1,T2,R1
			Basic cut set matrix for planarnetworks	**Define** cut set matrix	T1,T2,R1
			Basic tie set matrix for planarnetworks	**Define tie set** matrix	T1,T2,R1
	2.		Magnetic Circuits	**Understanding** Magnetic Circuits	T1,T2,R1
			Self and Mutual inductances	**Understanding** mutual inductance	T1,T2,R1
			Dot convention	**Know** about Dotconvention	T1,T2,R1
			Impedance	**Analyse** impedance	T1,T2,R1
			reactance concept	**Know** about reactance	T1,T2,R1
	3.		Impedance transformation andcoupled circuits	**Understanding** coupled circuits	T1,T2,R1
			Co-efficient of coupling	**Know** about coupling	T1,T2,R1
			Equivalent T for Magneticallycoupled circuits	**Understanding** Equivalent T Circuit	T1,T2,R1
			Ideal Transformer	**Know** about Ideal Transformer	T1,T2,R1
	4.	II.	Steady state and transient analysis ofRC Circuits	**Know** about Steadystate and transient	T1,T2,R1

				response	
			transient analysis of RL and RLC Circuits	**Analyse** transient analysis	T1,T2,R1
			transient analysis of RLC Circuits	**Analyse** transient analysis	T1,T2,R1
			Circuits with switches	**Understanding** switch circuit	T1,T2,R1
	5.		Step response	**Analyse** step response	T1,T2,R1
			2nd order series and parallel RLC Circuits	**Know** about 2ndorder Circuits	T1,T2,R1
			Root locus	**Understanding** Rootlocus	T1,T2,R1
			damping factor, over damped, underdamped	**Analyse** damped response	T1,T2,R1
	6.		Under damped, critically dampedcases,	**Analyse** damped response	T1,T2,R1
			Resonance curves	**Know** about resonance	T1,T2,R1
			Quality factor and bandwidth forseries	**Analyse** series resonance circuit	T1,T2,R1
			Quality factor and BW for parallel resonance	**Analyse** parallel resonance circuit	T1,T2,R1
			Two port network Z parameters	**Know about** Zparameters	T1,T2,R3
			Two port network using Y parameters	**Know about** Yparameters	T1,T2,R3
	7.		Two port network using ABCDparameters	**Know about** ABCD parameter	T1,T2,R3
			Two port network using h parameters	**Know about** hparameter	T1,T2,R3
			Two port network using g parameters	**Know about** gparameter	
	8.	III.	Characteristic impedance	**Define** Characteristic impedance	T1,T2,R3
			Image transfer constant	**Define** Image transferconstant	T1,T2,R3
			Image impedance	**Define** Image impedance	T1,T2,R3
			iterative impedance	**Define** iterative impedance	T1,T2,R2

	9.		network function	**Know about** networkfunction	T1,T2,R2
			Driving point transfer functions –using transformed (S) variables	**Know about** Drivingpoint transfer functions	T1,T2,R2

			Poles and Zeros	**Define** Poles andZeros	T1,T2,R2
	10		Standard T, π Sections	**Know about** StandardT, π Sections	T1,T2,R2
			Standard L Section	**Know about** Standard L Sections	T1,T2,R2
			Characteristic impedance	**Understanding** impedance	T1,T2,R2
			image transfer constants	**Understanding** imageconstants	T1,T2,R2
	11		Design of Attenuators	**Synthesis** Attenuators	T1,T2,R2
			impedance matching network	**Synthesis** impedancematching n/w	T1,T2,R2
			T and π Conversion,	**Understanding** T and π Conversion	T1,T2,R2
			LC Networks and Filters	**Synthesis** filter	T1,T2,R2
	12		constant K HP Filters	**Know about** constantK Filters	T1,T2,R2
			Design constant K HP Filters	**Synthesis** HP Filters	T1,T2,R2
			constant K BP Filters	**Know about** constantK Filters	T1,T2,R2
			Design constant K BP Filters	**Synthesis** BP Filters	T1,T2,R2
			Composite filter design.	**Synthesis** compositeFilters	T1,T2,R2
	13.	IV	**Transmission Lines - I** : Transmission line Types, Parameters, Transmission Line Equations		T1,T2,R2
			Transmission Lines - I : Transmission line Types, Parameters, Transmission Line Equations Primary & Secondary Constants	**Understanding**	T1,T2,R2
			Primary & Secondary Constants	**Gathering** Knowledge	T1,T2,R2
			Primary & Secondary Constants	**Gathering** Knowledge	T1,T2,R2
	14.		Expressions for Characteristic Impedance, Propagation Constant, Phase and Group Velocities	**Understanding of** Characteristic Impedance, Propagation Constant,Phase and Group	T1,T2,R2

				Velocities	
			Expressions for Characteristic Impedance, Propagation Constant, Phase and Group Velocities.	**Compose** th eKnowledge	T1,T2,R2
			Expressions for Characteristic Impedance, Propagation Constant,Phase and Group Velocities,	**Compose** th eKnowledge	T1,T2,R2
			Infinite Line Concepts Losslessness /Low Loss Characterization	**Compose** th eKnowledge	T1,T2,R2
	15.		Infinite Line Concepts Losslessness /Low Loss Characterization	**Gathering** Knowledge	T1,T2,R2
			Distortion – Condition for Distortion lessness Minimum Attenuation, Loading - Types of Loading.	**Gathering** Knowledge	T1,T2,R2
			Distortion – Condition for Distortion lessness Minimum Attenuation, Loading - Types of Loading.	**Compose** th eKnowledge	T1,T2,R2
	16	V	**Transmission Lines – II:** InputImpedance Relations		T1,T2,R1
			SC and OC Lines, Reflection Coefficient, VSWR	Knowledge of Reflection Coefficient, VSWR	T1,T2,R1
			SC and OC Lines, Reflection Coefficient, VSWR	**Gathering** Knowledge	T1,T2,R1
			Tutorial / Bridge Class # 9	**Understanding**	T1,T2,R1
	17		UHF Lines as Circuit Elements; λ /4,λ /2, λ /8 Lines	**Understanding**	T1,T2,R1
			UHF Lines as Circuit Elements; λ /4,λ /2, λ /8 Lines	**Gathering** Knowledge	T1,T2,R1

			Impedance Transformations	**Gathering** Knowledge	T1,T2,R1
			Significance of Z_{min} and Z_{max}	**Compose** th eKnowledge	T1,T2,R1
			Tutorial / Bridge Class # 10	**Gathering** Knowledge	T1,T2,R1
	18		Smith Chart – Configuration andApplications	**Understanding**	T1,T2,R1
			Smith Chart – Configuration andApplications	**Gathering** Knowledge	T1,T2,R1
			Single Stub Matching,	**Gathering** Knowledge	T1,T2,R1

6.UNIT WISE LECTURE NOTES
a)NotesofUnits

NETWORK ANALYSIS AND TRANSMISSION LINES

II B.Tech I semester(JNTUH-R18)

ELECTRONICS & COMMUNICATION ENGINEERING

UNIT – I:

Transient Analysis (First and Second Order Circuits):

- Introduction to transient response and steady state response
- **Transient response of series –RL, RC RLC Circuits for sinusoidal,square, ramp and pulse excitations**
- Initial Conditions
- **Solution using Differential Equations approach and Laplace Transform method**

Introduction to transient response and steady state response

- **In this chapter we shall study transient response of the RL, RC series and RLC circuits with sinusoidal, square, ramp and pulse excitations.**
- Transients are present in the circuit, when the circuit is subjected to any changes either by changing source magnitude or while changing any circuit elements, provided circuit consists of any energy storage elements.
- **There are 3 circuit elements(1)Resistor (2)Inductor(3)Capacitor**
- Inductor and Capacitor are called storage elements.
- **Inductor doesn't allow sudden change in current and stores the energy in the form of magnetic field.**
- Capacitor doesn't allow sudden change in voltage and stores the energy in the form of electric field.
- When the circuit is having only resistive elements, no transients present in the circuit since resistor allowssudden change in current and voltage and it doesn't store any energy.
- The total response of the circuit=Transient response +Steady state response.
- **Transient response changes with time and gets saturated after some time. It is also called as natural response.**
- **Steady state response** doesn't change with the time. It is also called forced response.
- **The time taken for the circuit to change from one steady state to another steady state is called transient time.**
- Under initial conditions inductor behaves like open circuit i.e. $I_L=0$
- **Under steady state conditions inductor behaves like short circuit i.e. $V_L=0$**
- Under initial conditions Capacitor behaves like short circuit i.e. $V_C=0$
- **Under steady state conditions capacitor behaves like open circuit i.e. $I_C=0$**

$t=0^-$ indicates immediately before operating switch

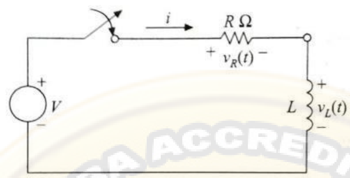


Fig1.1

$t=0^+$ indicates immediately after operating switch

$t=\infty$ indicates steady state condition

$t=0^-$ $\quad i_L=0$

$t=0^+$ $\quad i_L=0$

$t=\infty$ $\quad i_L=V/R$

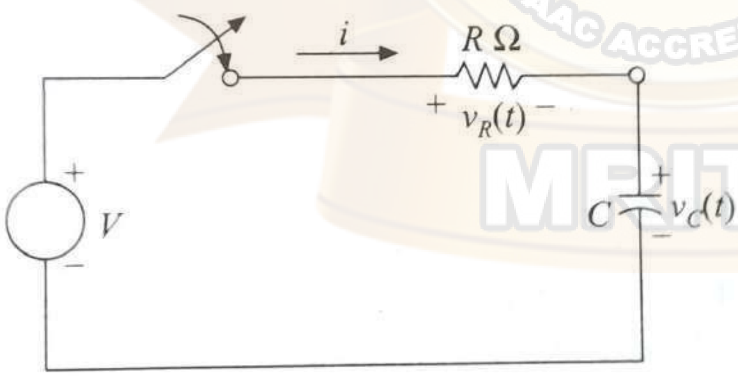


Fig1.2

$t=0^-$ $\quad V_c=0$

$t=0^+$ $\quad V_c=0$

The complementary function of the solution is

$t=\infty \quad V_c=V$

Transient response of series –RL Circuit for sinusoidal excitation

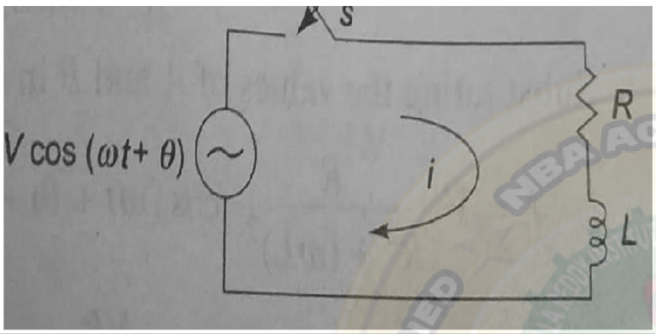


Fig1.3

Consider a circuit consisting of Series resistance and inductance as shown in fig1.3.The switch S is closed at t=0.

At t =0,a sinusoidal voltage V cos($\omega t+\theta$) is applied to the series RL circuit,where V is amplitude of the wave and θ is phase angle.

Application of KVL to the circuit results in the following differential equation.

$$V\cos(\omega \mathbf{t}+\theta)=Ri+L\frac{di}{dt} \text{--------------} \quad (1.1)$$

$$\frac{di}{dt}+\frac{R}{L}i=\frac{V}{L}\cos(\omega t+\theta)$$

The corresponding characteristic equation is

$$\left(D+\frac{R}{L}\right)i=\frac{V}{L}\cos(\omega t+\theta) \text{..................} (1.2)$$

For the above equation, the solution consists of two parts, viz.complementary function and particular integral.

The complementary function of the solution is

$$i_c = ce^{-t(R/L)}$$ -----(1.3)

The particular integral can be determined by using undetermined coefficients.

By assuming

$$i_p = A\cos(\omega t + \theta) + B\sin(\omega t + \theta)$$ ....(1.4)

$$i'_p = -A\omega\sin(\omega t + \theta) + B\omega\cos(\omega t + \theta)$$ ....(1.5)

Substituting equations (1.4) and (1.5) in equation (2)

$$\{-A\omega\sin(\omega t + \theta) + B\omega\cos(\omega t + \theta)\} + \frac{R}{L}\{A\cos(\omega t + \theta) + B\sin(\omega t + \theta)\} = \frac{V}{L}\cos(\omega t + \theta)$$

$$\left(-A\omega + \frac{BR}{L}\right)\sin(\omega t + \theta) + \left(B\omega + \frac{AR}{L}\right)\cos(\omega t + \theta) = \frac{V}{L}\cos(\omega t + \theta)$$

Comparing cosine terms and sine terms, we get

$$-A\omega + \frac{BR}{L} = 0$$

$$B\omega + \frac{AR}{L} = \frac{V}{L}$$

From the above equations, we have

$$A = V\frac{R}{R^2 + (\omega L)^2}$$

$$B = V\frac{\omega L}{R^2 + (\omega L)^2}$$

Substituting the values of A and B in equ(1.4),we get

$$i_p = V\frac{R}{R^2 + (\omega L)^2}\cos(\omega t + \theta) + V\frac{\omega L}{R^2 + (\omega L)^2}\sin(\omega t + \theta)$$

Putting $\quad M\cos\phi = \dfrac{VR}{R^2 + (\omega L)^2}$

and $\quad M\sin\phi = V\dfrac{\omega L}{R^2 + (\omega L)^2}$.

To find M and Φ,We divide one equation by the other

$$\frac{M\sin\phi}{M\cos\phi} = \tan\phi = \frac{\omega L}{R}$$

Squaring both equations and adding,we get

$$M^2\cos^2\phi + M^2\sin^2\phi = \frac{V^2}{R^2 + (\omega L)^2}$$

$$M = \frac{V}{\sqrt{R^2 + (\omega L)^2}}$$

The particular current becomes

$$i_p = \frac{V}{\sqrt{R^2 + (\omega L)^2}}\cos\left(\omega t + \theta - \tan^{-1}\frac{\omega L}{R}\right) \quad \text{..............(1.6)}$$

The complete solution for the current $i = i_c + i_p$

$$i = ce^{-t(R/L)} + \frac{V}{\sqrt{R^2 + (\omega L)^2}}\cos\left(\omega t + \theta - \tan^{-1}\frac{\omega L}{R}\right)$$

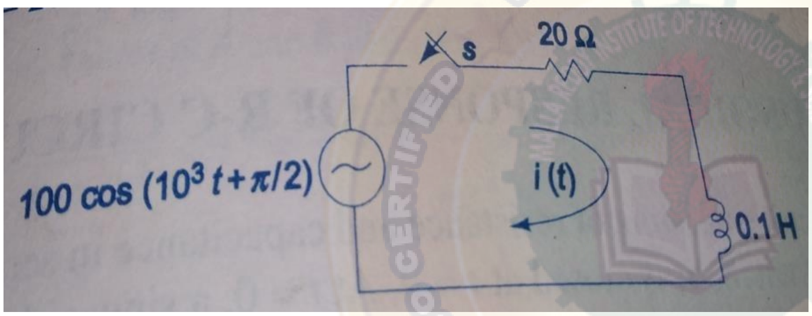
Since the inductor does not allow sudden change in currents, at t=0, i=0

$$c = -\frac{V}{\sqrt{R^2 + (\omega L)^2}}\cos\left(\theta - \tan^{-1}\frac{\omega L}{R}\right)$$

$$i = e^{-(R/L)t}\left[\frac{-V}{\sqrt{R^2+(\omega L)^2}}\cos\left(\theta-\tan^{-1}\frac{\omega L}{R}\right)\right] + \frac{V}{\sqrt{R^2+(\omega L)^2}}\cos\left(\omega t+\theta-\tan^{-1}\frac{\omega L}{R}\right)$$

Example1.1

In the circuit as shown in figure below, determine the complete solution for the current, when switch S is closed at t=0.Applied voltage v(t)=100cos(10^3t+π/2).Resistance R=20Ω and inductance L=0.1H.



Solution

By applying Kirchhoff's voltage law to the circuit, we have

$20i+0.1\frac{di}{dt}=100\cos(10^3t+\pi/2)$.

$\frac{di}{dt}+200i=1000\cos(1000t+\pi/2)$

$(D+200)i=1000\cos(1000t+\pi/2)$

The complementary function $i_c=ce^{-200t}$

By assuming particular integral as

$i_p=A\cos(\omega t+\theta)+B\sin(\omega t+\theta)$

We get

$i_p = \frac{V}{\sqrt{R^2+(\omega \mathbf{L})^2}} \cos(\omega \mathbf{t} + \theta - \tan^{-1}\frac{\omega \mathbf{L}}{R})$

Where $\omega = \mathbf{1000\ rad/sec}$

V=100 V,$\theta = \frac{\pi}{2}$

L=0.1H,R=20Ω

Substituting the values in the above equation, we get

$i_p = \frac{100}{\sqrt{20^2+(\mathbf{1000*0.1})^2}} \cos(\mathbf{1000t} + \frac{\pi}{2} - \tan^{-1}\frac{\mathbf{100}}{20})$

$= \frac{100}{101.9} \cos(1000t + \frac{\pi}{2} - 78.6°)$

$=0.98 \cos(1000t + \frac{\pi}{2} - 78.6°)$

The complete solution is

$i = ce^{-200t} + 0.98\cos(1000t + \frac{\pi}{2} - 78.6°)$

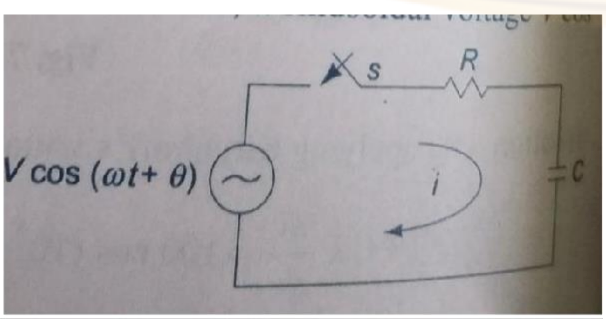
At t=0, the current flowing through the circuit is zero,.i.e.i=0

$c = -0.98\cos(\frac{\pi}{2} - 78.6°)$

The complete solution is

$i = [-0.98\cos(\frac{\pi}{2} - 78.6°)]\, e^{-200t} + 0.98\cos(1000t + \frac{\pi}{2} - 78.6°)]$

<u>SINUSOIDAL RESPONSE OF R-C CIRCUIT:</u>



Consider a circuit consisting of resistance and capacitance in series as shown in fig. The switch, S,is closed at t=0.At t=0,a sinusoidal voltage $V\cos(\omega t+\theta)$ is applied to the R-C circuit,where V is the amplitude of the wave and θ =Phase angle.

Applying KVL to the circuit results in the following differential equation.

$V\cos(\omega t+\theta)=Ri+\frac{1}{C}\int idt$ ---------------(1.7)

$$R\frac{di}{dt}+\frac{i}{C}=-V\omega(\sin\omega t+\theta)$$

$$(D+\frac{1}{RC})i=\frac{-V\omega}{R}(\sin\omega t+\theta) \quad (1.8)$$

The complementary function $i_c=Ke^{-t/RC}$--------------------(1.9)

The particular solution can be obtained by using undetermined coefficients.

$$i_p=A\cos(\omega t+\theta)+B\sin\omega t+\theta) \quad (1.10)$$

$ip^1=-A\omega\sin(\omega t+\theta)+B\omega\cos(\omega t+\theta)$----------------(1.11)

Substituting equations 1.10 and 1.11 in 1.8 we get

$$\{-A\omega\sin(\omega t+\theta)+B\omega\cos(\omega t+\theta)\}+\frac{1}{RC}A\cos(\omega t+\theta)+B\sin\omega t+\theta)=\frac{-V\omega\sin\omega t+\theta)}{R}$$

Comparing both sides

$-A\omega+\frac{B}{RC}=\frac{-V\omega}{R}$

$B\omega+\frac{A}{RC}=0$

From which,

$A=\frac{VR}{R^2+(\frac{1}{mc})^2}$

$$B = -\frac{V}{\omega C\left(R^2+\left(\frac{1}{\omega C}\right)^2\right)}$$

Substituting values of A and B in equation (1.10), we have

$$i_p = \frac{VR}{R^2+\left(\frac{1}{\omega C}\right)^2}\cos(\omega t+\theta) + -\frac{V}{\omega C\left(R^2+\left(\frac{1}{\omega C}\right)^2\right)}\sin(\omega t+\theta)$$

Putting

$$M\cos\emptyset = \frac{VR}{R^2+\left(\frac{1}{\omega C}\right)^2}$$

$$M\sin\emptyset = \frac{V}{\omega C\left(R^2+\left(\frac{1}{\omega C}\right)^2\right)}$$

To find out M and Ø, we divide one equation by other,

$$\frac{M\cos\emptyset}{M\sin\emptyset} = \tan\emptyset = \frac{1}{\omega CR}$$

Squaring both sides and adding, we get

$$(M\cos\emptyset)^2 + (M\sin\emptyset)^2 = \frac{V^2}{\left(R^2+\left(\frac{1}{\omega C}\right)^2\right)}$$

$$M = \frac{V}{\sqrt{\left(R^2+\left(\frac{1}{\omega C}\right)^2\right)}}$$

The particular current becomes

$$i_p = \frac{V}{\sqrt{\left(R^2+\left(\frac{1}{\omega C}\right)^2\right)}}\cos\left(\omega t+\theta+\tan^{-1}\frac{1}{\omega CR}\right)$$

)------------- **(1.1 2)**

$\overline{\omega}$**c**

The complete solution for the current $i=i_c+i_p$

$$i = Ke^{-t/RC} + \frac{V}{\sqrt{R^2+\left(\frac{1}{\omega c}\right)^2}}\cos\left(\omega t+\theta+\tan^{-1}\frac{1}{\omega CR}\right) \text{------------- (1.13)}$$

Since the capacitor does not allow sudden change in voltages at $t=0, i=$ $\frac{V}{R}\cos\theta$

$$\frac{V}{R}\cos\theta = K + \frac{V}{\sqrt{R^2+\left(\frac{1}{\omega c}\right)^2}}\cos\left(\theta+\tan^{-1}\frac{1}{\omega CR}\right)$$

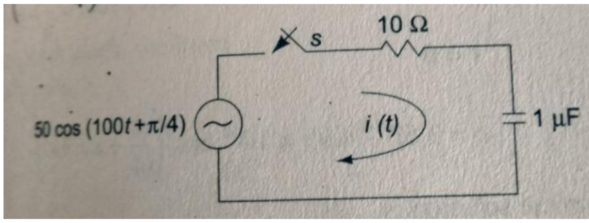
$$K = \frac{V}{R}\cos\theta - \frac{V}{\sqrt{R^2+\left(\frac{1}{\omega c}\right)^2}}\cos\left(\theta+\tan^{-1}\frac{1}{\omega CR}\right)$$

The complete solution for the current is

$$I = e^{-\frac{t}{RC}}\left[\frac{V}{R}\cos\theta - \frac{V}{\sqrt{R^2+\left(\frac{1}{\omega c}\right)^2}}\cos\left(\theta+\tan^{-1}\frac{1}{\omega CR}\right)\right] + \frac{V}{\sqrt{R^2+\left(\frac{1}{\omega c}\right)^2}}\cos\left(\theta+\tan^{-1}\frac{1}{\omega CR}\right) \text{------------- (1.14)}$$

Example 1.2.

In the circuit as shown in Figure below, determine the complete solution for the current when switch S is closed at t=0.Applied voltage is $v(t)=50\cos(10^2t+\pi/4)$.Resistance R=10Ω and capacitance $C=1\mu F$.


s
10 Ω
50 cos (100t + π/4)
i (t)
1 µF

Solution:

By applying KVL to the circuit, we have

$$\mathbf{10}i+\frac{1}{10^{-6}}\int idt=\mathbf{50}\cos(\mathbf{100t}+\boldsymbol{\pi/4})$$

$$\mathbf{10}\frac{di}{dt}+\frac{i}{10^{-6}}=-5\times10^{3}(\mathbf{\sin 100t}+\boldsymbol{\pi/4})$$

$$\frac{di}{dt}+\frac{i}{10^{-5}}=500(\mathbf{\sin 100t}+\boldsymbol{\pi/4})$$

$$(D+\frac{1}{10^{-5}})i=-500(\mathbf{\sin 100t}+\boldsymbol{\pi/4})$$

The complementary function $i_C=Ke^{-t/10^{-5}}$

The particular solution $i_p=A\cos(\omega t+\theta)+B\sin(\omega t+\theta)$

We get $i_p=\dfrac{V}{\sqrt{R^2+(\frac{1}{\omega c})^2}}\cos\left(\omega t+\theta+\tan^{-1}\frac{1}{\omega CR}\right)$

Where $\omega=100\frac{rad}{sec}$ $\quad\theta=\frac{\pi}{4}$

R=10Ω C=1μF

$$i_p=\frac{500}{\sqrt{(10^2+(\frac{1}{100\times10^{-6}})^2)}}\cos\left(100t+\frac{\pi}{4}+\tan^{-1}\frac{1}{100\times10^{-6}\times10}\right)$$

$$i_p=4.99\times10^{-3}\cos\left(100t+\frac{\pi}{4}+89.94°\right)$$

$\underline{\textbf{at t= 0,} \boldsymbol{i} =} \ \frac{V}{R}\cos\theta = \frac{50}{10}\cos\frac{\pi}{4} \underline{\mathbf{= 3.53}\ \boldsymbol{A}}$

i= $Ke^{-t/10^{-5}}$ $\underline{\mathbf{+\,4.99 \times 10^{-3}}}$ $\mathbf{cos}\,(100\mathbf{t} + \frac{\pi}{4}\underline{\mathbf{+\,89.94°)}}$

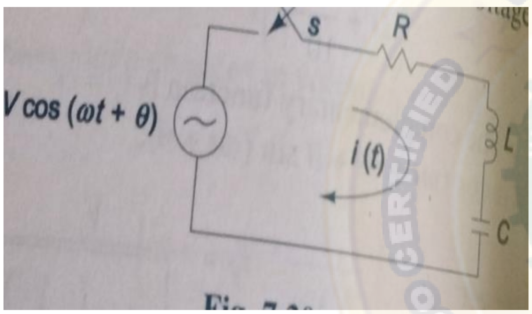
<u>At t=0</u>

$$K=3.53-\mathbf{4.99}\times 10^{-3}\,\mathbf{cos}\left(\frac{\pi}{4}+\mathbf{89.94°}\right)$$

<u>Hence the complete solution is</u>

$$i=\left[\mathbf{3.53}-\mathbf{4.99}\times 10^{-3}\,\mathbf{cos}\left(\frac{\pi}{4}+\mathbf{89.94°}\right)\right]\boldsymbol{e}^{-\boldsymbol{t}/10^{-5}}+\mathbf{4.99}\times 10^{-3}\,\mathbf{cos}\left(100\mathbf{t}+\frac{\pi}{4}+\mathbf{89.94°}\right)$$

<u>SINUSOIDAL RESPONSE OF RLC CIRCUIT:</u>



Consider a circuit consisting of resistance, inductance and capacitance in series as shown in fig. The switch, S is closed at t=0.At t=0,a sinusoidal voltage $V\cos(\omega\mathbf{t}+\theta)$ is applied to the RLC series circuit ,where V is the amplitude of the wave and θ =Phase angle.

Applying KVL to the circuit results in the following differential equation.

$$\mathbf{V\cos(\omega t+\theta)=RI+L}\frac{di}{dt}+\frac{1}{C}\int i\,dt \text{ --------- } \mathbf{(1.15)}$$

<u>Differentiating above equation, we get</u>

$$R\frac{di}{dt}+L\frac{d^2i}{dt^2}+\frac{i}{C}=-V\omega\sin(\omega\mathbf{t}+\theta)$$

$$\left(D^2+\frac{R}{L}D+\frac{1}{LC}\right)i=-\frac{V\omega}{L}\,\mathbf{\sin(\omega t+\theta)} \text{ ------------------ } \mathbf{(1.16)}$$

The particular solution can be obtained by using undetermined coefficients.

$i_p = A\cos(\omega t + \theta) + B\sin(\omega t + \theta)$-----------(1.17)

$i_p^1 = -A\omega \sin(\omega t + \theta) + B\omega \cos(\omega t + \theta)$---------------------------- (1.18)

$$ip'' = -A\omega^2 \cos(\omega t + \theta) - B\omega^2 \sin(\omega t + \theta) \quad (1.19)$$

Substituting values of i_p, ip^1, ip'' in equ (1.16) we have

$$-A\omega^2 \cos(\omega t + \theta) - B\omega^2 \sin(\omega t + \theta) + \frac{R}{L}[-A\omega \sin(\omega t + \theta) + B\omega \cos(\omega t + \theta)] +$$

$$\frac{1}{LC}[A\cos(\omega t + \theta) + B\sin(\omega t + \theta)] = -\frac{V\omega}{L}\sin(\omega t + \theta) \quad (1.20)$$

Comparing both sides, we haveSine coefficients

$$-B\omega^2 - A\omega\frac{R}{L} + \frac{B}{LC} = -\frac{V\omega}{L}$$

$$-A\frac{\omega R}{L} + B\left(\omega^2 - \frac{1}{LC}\right) = \frac{V\omega}{L} \text{------------- } (1.21)$$

Cosine coefficients

$$-A\omega^2 + B\omega\frac{R}{L} + \frac{A}{LC} = 0$$

$$A\left(\omega^2 - \frac{1}{LC}\right) - B\left(\frac{\omega R}{L}\right) = 0 \text{-------------- } (1.22)$$

Solving (1.21) and(1.22) we get

$$A = \frac{\frac{V\omega^2 R}{L^2}}{\left[\left(\frac{\omega R}{L}\right)^2 - \left(\left(\omega^2 - \frac{1}{LC}\right)^2\right)\right]}$$

$$B = \frac{(\omega^2 - \frac{1}{LC})^2 V\omega}{L[(\frac{\omega R}{L})^2 - ((\omega^2 - \frac{1}{LC})^2)]}$$

Substituting values of A and B in equation (1.17),We get

$$i_p = \frac{\frac{V\omega^2 R}{L}}{[(\frac{\omega R}{L})^2 - ((\omega^2 - \frac{1}{LC})^2)]}\cos(\omega t + \theta) + \frac{(\omega^2 - \frac{1}{LC})^2 V\omega}{L[(\frac{\omega R}{L})^2 - ((\omega^2 - \frac{1}{LC})^2)]}\sin(\omega t + \theta) \text{ --------------- } (1.23)$$

Putting

$$M\cos\emptyset = \frac{\frac{V\omega^2 R}{L^2}}{[(\frac{\omega R}{L})^2 - ((\omega^2 - \frac{1}{LC})^2)]}$$

$$M\sin\emptyset = \frac{(\omega^2 - \frac{1}{LC})^2 V\omega}{L[(\frac{\omega R}{L})^2 - ((\omega^2 - \frac{1}{LC})^2)]}$$

To find out M and Ø, we divide one equation by other,

$$\frac{M\cos\emptyset}{M\sin\emptyset} = \tan\emptyset = \frac{(\omega L - \frac{1}{\omega C})}{R}$$

$$\emptyset = \tan^{-1}\left[\frac{(\omega L - \frac{1}{\omega C})}{R}\right]$$

Squaring both equations and adding we get

$$(M\cos\emptyset)^2 + (M\sin\emptyset)^2 = \frac{V^2}{(R^2 + (\frac{1}{mc} - \omega L)^2)}$$

$$M = \frac{V}{\sqrt{(R^2 + (\frac{1}{mc} - \omega L)^2)}}$$

The particular current becomes

$$i_p = \frac{V}{\sqrt{(R^2 + (\frac{1}{mc} - \omega L)^2)}} \cos\left(\omega t + \theta + \tan^{-1}\left[\frac{(\omega L - \frac{1}{\omega C})}{R}\right]\right)$$

$$\sqrt{(R^2+(\frac{1}{\omega c}-\omega L)^2)} \quad \textbf{(1.24)}$$

To find out complementary function ,we have the characteristic equation

$(D^2 + \frac{R}{L}D + \frac{1}{LC}) = 0$------------- (1.25)

The roots of equation(1.25) are

$$D_1, D_2 = -\frac{R}{2L} \pm \sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{LC}}$$

By assuming $K_1 = -\frac{R}{2L}$

$$K_2 = \sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{LC}}$$

$D_1 = K_1 + K_2$

$D_1 = K_1 - K_2$

K_2 becomes positive, when $\left(\frac{R}{2L}\right)^2 > \frac{1}{LC}$

The roots are real and unequal, which gives an over damped response. Then equation (1.25) becomes

$$[D - (K1 + K2)][D - (K1 - K2)]i = 0$$

The complementary function of above equation is

$$ic = c_1 e^{(k1+k2)t} + c2e^{(k1-k2)t} + \frac{V}{\sqrt{(R^2 + (\frac{1}{\omega c} - \omega L)^2)}} \cos\left(\omega t + \theta + \tan^{-1}\left[\frac{(\omega L - \frac{1}{\omega C})}{R}\right]\right)$$

K2 becomes negative when $\left(\frac{R}{2L}\right)^2 < \frac{1}{LC}$

$2L$ LC

Then the roots are complex conjugate, which gives an under damped response.

Then equation (1.25) becomes

$$[D - (\mathrm{K1} + j\mathrm{K2})][D - (\mathrm{K1} - j\mathrm{K2})]i = 0$$

The solution for above equation is

$i_c = e^{k1t}[c1\cos k2t + c2\sin k2t]$

i=ic+ip

$i = e^{k1t}[c1\cos k2t + c2\sin k2t] + \frac{V}{\sqrt{(R^2 + (\frac{1}{\omega c} - \omega L)^2)}}\cos\left(\omega t + \theta + \tan^{-1}\left[\frac{(\omega L - \frac{1}{\omega C})}{R}\right]\right)$

k2 becomes zero when $\left(\frac{R}{2L}\right)^2 = \frac{1}{LC}$

Then the roots are equal which gives critically damped response Then equation (1.25) becomes $(D - K1)(D - K1)i = 0$

The complementary function for the above equation is

$ic = e^{(k1)t}[c1 + c2t]$

Therefore complete solution is i=ic+ip

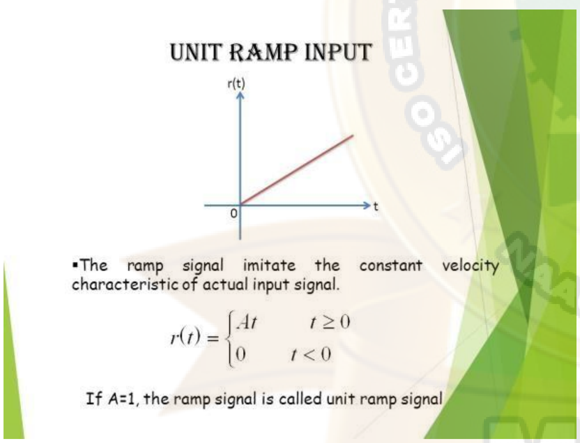
$$e^{(k1)t}[c1 + c2t] + \frac{V}{\sqrt{(R^2 + (\frac{1}{\omega c} - \omega L)^2)}}\cos\left(\omega t + \theta + \tan^{-1}\left[\frac{(\frac{1}{\omega C} - \omega L)}{R}\right]\right)$$

$$F(s) = \mathcal{L}\{f(t)\} = \int_0^\infty e^{-st} f(t)\,dt$$

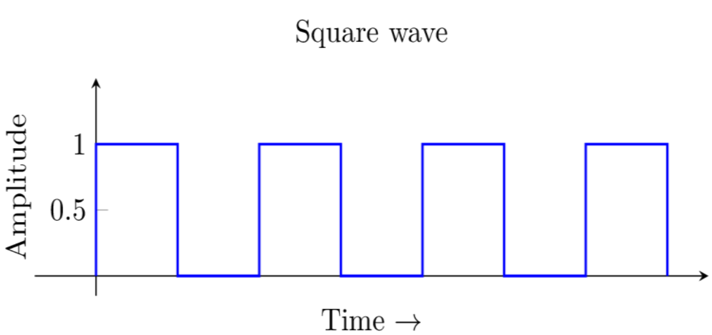
Solution using Laplace transformation method:

$f(t)$ (Function)	$F(s)$ (Laplace Transform)
$u(t)$ (unit step)	$1/s$
$\delta(t)$ (unit impulse)	1
e^{-at}	$\frac{1}{(s+a)}$
$\sin \omega t$	$\frac{w}{(s^2+\omega^2)}$
$\cos \omega t$	$\frac{s}{(s^2+\omega^2)}$
$e^{-at} \sin \omega t$	$\frac{\omega}{(s+a)^2+\omega^2}$
$e^{-at} \cos \omega t$	$\frac{(s+a)}{(s+a)^2+\omega^2}$
t	$1/s^2$
$\frac{df(t)}{dt}$	$sF(s)$
$\int f(t)dt$	$F(s)/s$

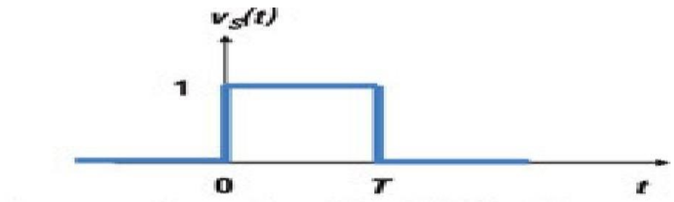
Ramp input



Square input



Pulse input



Vs(t)

V_s(t)=u(t)-u(t-T)

$V_s(s) = \frac{1-e^{-ST}}{S}$

UNIT – II:

Two Port Networks:

- **Impedance Parameters,**
- Admittance Parameters,
- **Hybrid Parameters,**
- Transmission (ABCD) Parameters,
- **Conversion of one of parameter to another,**
- Conditions for Reciprocity and Symmetry,
- **Interconnection of two port networks in Series, Parallel and Cascaded configurations, Image Parameters,**
- Illustrative problems.

Introduction:

A general network having two pairs of terminals, one labeled the “input terminals’’ and the other the “output terminals,’’ is a very important building block in electronic systems, communication systems, automatic control systems, transmission and distribution systems, or other systems in which an electrical signal or electric energy enters the input terminals, is acted upon by the network, and leaves via the output terminals. A pair of terminals at which a signal may enter or leave a network is also called a **port,** and a network like the above having two such pair of terminals is called a **Two - port network.** A general two-port network with terminal voltages and currents specified is shown in the figure below. In such networks the relation between the two voltages and the two currents can be described in six different ways resulting in six different systems of Parameters and in this chapter we will consider the most important four systems

Impedance Parameters: Z parameters (open circuit impedance parameters)

We will assume that the two port networks that we will consider are composed of linear elements and contain no independent sources but dependent sources *are* permissible. We will consider the two-port networks shown in the figure below.

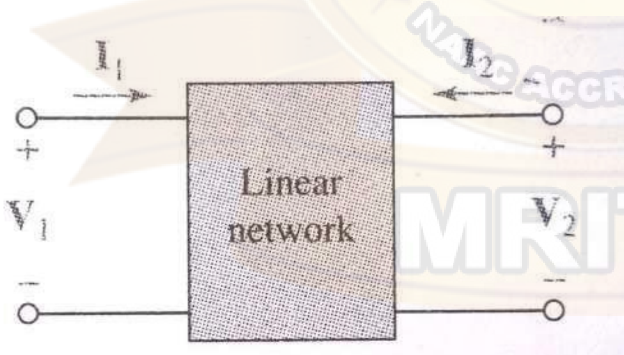


Fig 5.1: Ageneraltwo-portnetworkwithterminalvoltagesandcurrentsspecified.Thetwo- port network is composed of linear elements, possibly including dependent sources, but not containing any independent sources.

The voltage and current at the input terminals are $\mathbf{V}_1$ & $\mathbf{I}_1$, and $\mathbf{V}_2$ & $\mathbf{I}_2$ are voltage and current at the output port. The directions of $\mathbf{I}_1$ and $\mathbf{I}_2$ are both customarily selected as into the network at the upper conductors (and out at the lower conductors). Since the network is

Linear and containsnoindependent sources withinit, V_1 may beconsidered tobe the superpositionof two components,onecausedbyI_1 AndtheotherbyI_2.WhenthesameargumentisappliedtoV_2, we get the set of equations

$V_1 = Z_{11}I_1 + Z_{12}I_2$

$V_2 = Z_{21}I_1 + Z_{22}I_2$

[V] = [Z][I]

Where **[V],[Z] and [I]**areVoltage,impedanceandcurrentmatrices.Thedescriptionofthe**Z** parameters,definedin theabove equationsisobtainedbysettingeachofthecurrentsequalto zero as givenbelow.

$Z_{11}=V_1/I_1 \mid I_2=0$ $Z_{12}=V_1/I_2 \mid I_1=0$ $Z_{21}=V_2/I_1 \mid I_2=0$ $Z_{22}=V_2/I_2 \mid I_1=0$

Thus, since zero current results from an open-circuit termination, the **Z** parameters are known as the **Open-circuit Impedance parameters**. And more specifically Z_{11} & Z_{22} are called **Driving point Impedances** and Z_{12} & Z_{21} are called **Reverse and Forward transfer impedances** respectively. A basic Z parameter equivalent circuit depicting the above defining equations is shown in the figure below.

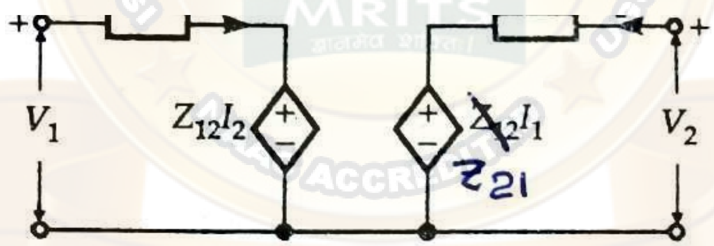


Fig 5.2: Z-Parameter equivalent circuit

Admittance parameters: (Y Parameters or Short circuit admittance parameters)

The same general two port network shown for **Z** parameters is applicable here also and is shown below.

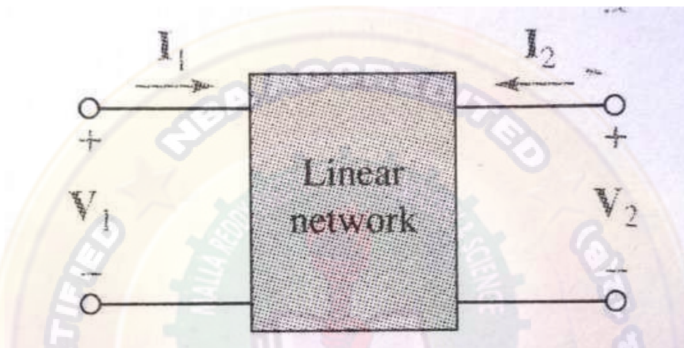


Fig 5.3: A general two-port network with terminal voltages and currents specified. The two- port network is composed of linear elements, possibly including dependent sources, but not containing any independent sources.

Since the network is linear and contains no independent sources within, on the same lines of **Z** parameters the defining equations for the Y parameters are given below. I_1 and I_2 may be considered to be the superposition of two components, one caused by V_1 and the other by V_2 and then we get the set of equations defining the **Y** parameters.

$$I_1 = Y_{11}V_1 + Y_{12}V_2$$

$$I_2 = Y_{21}V_1 + Y_{22}V_2$$

Where the **Y**s are no more than proportionality constants and their dimensions are A/V (Current/Voltage). Hence they are called the **Y** (or admittance) parameters. They are also defined in the matrix form given below.

And in much simpler form as

$$[I] = [Y][V]$$

The individual Y parameters are defined on the same lines as Z parameters but by setting either of the voltages $\mathbf{V_1}$ and $\mathbf{V_2}$ as zero as given below.

The most informative way to attach a physical meaning to the **y** parameters is through a direct inspection of defining equations. The conditions which must be applied to the basic defining equations are very important. In the first equation for example; if we let $\mathbf{V}_2$ zero, then $\mathbf{Y}_{11}$ is given by the ratio of $\mathbf{I}_1$ to $\mathbf{V}_1$. We therefore describe $\mathbf{Y}_{11}$ as the admittance measured at the input terminals with the output terminals *short-circuited* ($\mathbf{V}_2 = 0$). Each of the **Y** parameters may be described as a ***current-voltage*** ratio with either $V_1 = 0$ (the input terminals short circuited) or $V_2 = 0$ (the output terminals short-circuited):

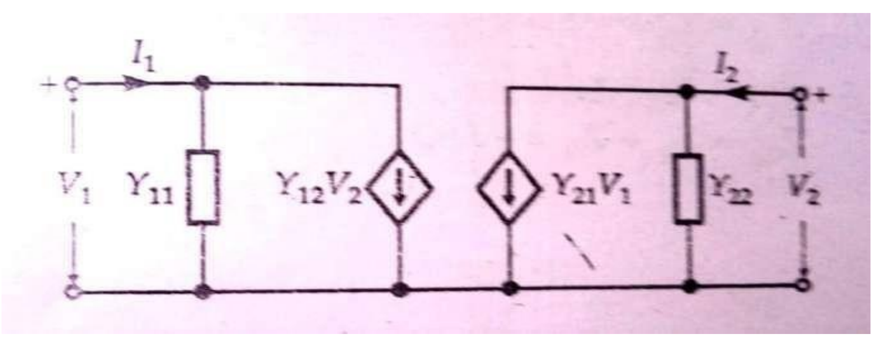
$$\mathbf{Y_{11} = I_1/V_1 \quad with \quad V_2 = 0}$$

$$\mathbf{Y_{12} = I_1/V_2 \quad with \quad V_1 = 0}$$

$$\mathbf{Y_{21} = I_2/V_1 \quad with \quad V_2 = 0}$$

$$\mathbf{y_{22} = I_2/V_2 \quad with \quad V_1 = 0}$$

Because each parameter is an ***admittance*** which is obtained by short circuiting either the output or the input port, the **Y** parameters are known as the ***short-circuit admittance parameters.*** The specific name of $\mathbf{Y}_{11}$ is the***short-circuit input admittance,*** $\mathbf{Y}_{22}$ is the ***short circuit output admittance,*** and $\mathbf{Y}_{12}$and $\mathbf{Y}_{21}$ are the ***short-circuit reverse and forward transfer admittances respectively.***



h parameter representation is used widely in modeling of Electronic components and circuits particularlyTransistors.Hereboth shortcircuitandopencircuitconditionsareutilized.

The hybrid parameters are defined by writing the pair of equations relating **V_1**, **I_1**, **V_2**, and **I_2**:

$$V_1 = h_{11}.I_1 + h_{12}.V_2$$

$$I_2 = h_{21}.I_1 + h_{22}.V_2$$

The nature of the parameters is made clear by first setting **$V_2 = 0$**. Thus,

$h_{11} = V_1/I_1$ with $V_2 = 0$ = ***short-circuit input impedance***

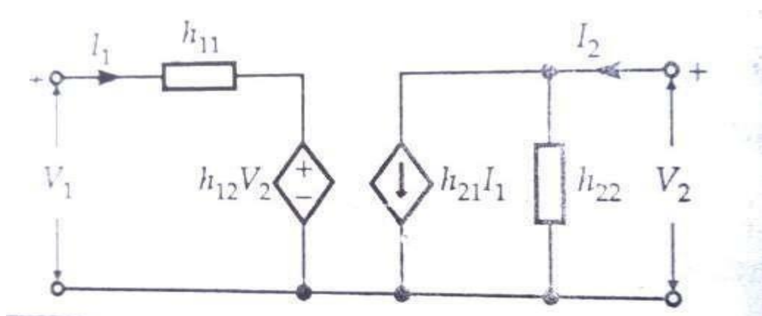
$h_{21} = I_2/I_1$ with $V_2 = 0$ = ***short-circuit forward current gain***

Then letting **$I_1 = 0$**, we obtain **$h_{12} = V_1/V_2$ with $I_1 = 0$** = ***open-circuit reverse voltage gain***

$h_{22} = I_2/V_2$ with $I_1 = 0$ = ***open-circuit output admittance***

Since the parameters represent an impedance, an admittance, a voltage gain, and a current gain, they are called the "hybrid'' parameters.

The subscript designations for these parameters are often simplified when they are applied to transistors. Thus, **h**$_{11}$, **h**$_{12}$, **h**$_{21}$, and **h**$_{22}$ become **h**$_i$, **h**$_r$, **h**$_f$, and **h**$_o$, respectively, where the subscripts denote input, reverse, forward, and output.



Transmission parameters:

The last two-port parameters that we will consider are called the **t parameters,** the **ABCD parameters,** orsimplythe **transmission parameters.** Theyaredefinedbytheequations

$$V_1 = A.V_2 - B.I_2$$

$$I_1 = C.V_2 - D.I_2$$

and in Matrix notation these equations can be written in the form

$$\begin{matrix} V_1 & = & A\ B & V_2 \\ I_1 & = & C\ D & -I_2 \end{matrix}$$

where $\mathbf{V}_1$, $\mathbf{V}_2$, $\mathbf{I}_1$, and $\mathbf{I}_2$ are defined as as shown in the figure below.

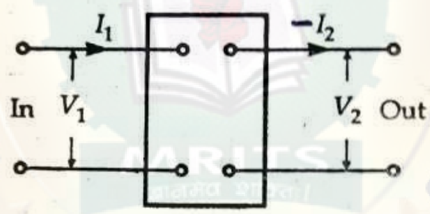


Fig 5.6: Two port Network for ABCD parameter representation with Input and output Voltages

andcurrents

The minus signs that appear in the above equations should be associated with the output current, as ($-\mathbf{I}_2$). Thus, both $\mathbf{I}_1$ and $-\mathbf{I}_2$ are directed to the right, the direction of energy or signal transmission.

Note that there are no minus signs in the **t** or **ABCD** matrices. Looking again at the above equations

we see that the quantities on the left, often thought of as the given or independent variables, are the input voltage and current, $\mathbf{V}_1$ and $\mathbf{I}_1$; the dependent variables, $\mathbf{V}_2$ and $\mathbf{I}_2$, are the output quantities. Thus, the transmission parameters provide a direct relationship between input and output. Their major use arises in transmission-line analysis and in cascadednetworks.

The four Transmission parameters are defined and explained below.

First **A** and **C** are defined with receiving end open circuited i.e. with $I_2 = 0$

$A = V_1/V_2$ with $I_2 = 0$ = Reverse voltage Ratio $C = I_1/V_2$ with $I_2 = 0$ = **Transfer admittance**

*Next **B** and **D** are defined with receiving end short circuited i.e. with $V_2 = 0$*

$B = V_1/{-I_2}$ with $V_2 = 0$ = Transfer impedance $D = I_1/{-I_2}$ with $V_2 = 0$ = Reverse current ratio

Inter relationships between different parameters of two port networks:

Basic Procedure for representing any of the above four two port Network parameters in terms of the other parameters consists of the following steps:

1. Write down the defining equations corresponding to the parameters in terms of which the other parametersare to be represented.
2. Keeping the basic parameters same, rewrite/manipulate these two equations in such a way that the variables V_1, V_2, I_1, and I_2 are arranged corresponding to the defining equations of the first parameters.
3. Then by comparing the parameter coefficients of the respective variables V_1, V_2, I_1, and I_2 on the right hand side of the two sets of equations we can get the interrelationship.

Z Parameters in terms of Y parameters:

Though this relationship can be obtained by the above steps, the following simpler method is used for Z in terms of Y and Y in terms of Z:

Z and Y being the Impedance and admittance parameters (Inverse), in matrix notation they are governed by the following inverse relationship.

$$[Z] = [Y]^{-1}$$

Or:

$$\begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix}^{-1}$$

Thus :

$$Z_{11} = \frac{Y_{22}}{\Delta Y}, Z_{12} = -\frac{Y_{12}}{\Delta Y}$$

$$Z_{21} = -\frac{Y_{21}}{\Delta Y} \text{ and } Z_{22} = \frac{Y_{11}}{\Delta Y}$$

$$\left[\text{Here } \Delta Y = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} = Y_{11} Y_{22} - Y_{12} Y_{21}\right]$$

$$V_2 = \frac{1}{C} \cdot I_1 + \frac{D}{C} \cdot I_2$$

Z Parameters in terms of ABCD parameters:

The governing equations are:

$V_1 = AV_2 - BI_2$

$I_1 = CV_2 - DI_2$

from the second governing equation [$I_1 = CV_2 - DI_2$] we canwrite

$$V_1 = \left[\frac{1}{C} \cdot I_1 + \frac{D}{C} \cdot I_2\right] A - BI_2$$

$$= \frac{A}{C} \cdot I_1 + \frac{AD - BC}{C} \cdot I_2$$

$$Z_{11} = \frac{A}{C}, Z_{12} = \frac{AD - BC}{C}$$

$$Z_{21} = \frac{1}{C}, Z_{22} = \frac{D}{C}$$

Now substituting this value of V_2 in the first governing equation [$V_1 = AV_2 - BI_2$] we get

Comparing these two equations for **V_1** and **V_2** with the governing equations of the **Z** parameter network we get **Z** Parameters in terms of ABCD parameters:

Z Parameters in terms of h parameters:

Thegoverningequationsofhparameternetworkare: $V_1 = h_{11}I_1 + h_{12}V_2$

$I_2 = h_{21} I_1 + h_{22} V_2$

From the second equation we get

$$V_2 = -\frac{h_{21}}{h_{22}} . I_1 + \frac{1}{h_{22}} . I_2$$

Substituting this value of V_2 in the first equation for V_1 we get:

$$V_1 = h_{11} I_1 + h_{12} V_2$$
$$= h_{11} I_1 + h_{12}\left[-\frac{h_{21}}{h_{22}} I_1 + \frac{1}{h_{22}} . I_2\right]$$
$$= \frac{\Delta h}{h_{22}} I_1 + \frac{h_{12}}{h_{22}} . I_2$$

Now comparing these two equations for V_1 and V_2 with the governing equations of the **Z** Parameter network we get **Z** Parameters in terms of **h** parameters:

$$Z_{11} = \frac{\Delta h}{h_{22}}, \quad Z_{12} = \frac{h_{12}}{h_{22}}$$
$$Z_{21} = -\frac{h_{21}}{h_{22}}, Z_{22} = \frac{1}{h_{22}}$$

Here $\Delta h = h_{11} h_{22} - h_{12} h_{21}$

Y Parameters in terms of Z parameters:

YandZbeingtheadmittanceandImpedanceparameters(Inverse),inmatrixnotationtheyare governed bythe following inverse relationship.

$$[Y] = [Z]^{-1}$$

Or:

$$\begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix}^{-1}$$

Thus:

$$Y_{11} = \frac{Z_{22}}{\Delta Z}, \quad Y_{12} = -\frac{Z_{12}}{\Delta Z}$$

$$Y_{21} = -\frac{Z_{21}}{\Delta Z}, \quad Y_{22} = \frac{Z_{11}}{\Delta Z}$$

Here $\Delta Z = Z_{11} Z_{22} - Z_{12} Z_{21}$

The other inter relationships also can be obtained on the same lines following the ***basic three steps given*** in thebeginning.

Conditions for reciprocity and symmetry in two port networks:

A two portnetworkis said to be ***reciprocal*** ifthe ratioof the output responsevariabletothe input excitation variableis samewhentheexcitationandresponseportsareinterchanged.

A two port network is said to be ***symmetrical*** if the port voltages and currents remain the same when the input and output ports are interchanged.

Inthistopicwewillgettheconditionsfor***Reciprocity*** and***symmetry*** forallthefournetworks. Thebasicprocedure foreachofthenetworksconsistsofthefollowingsteps:

Reciprocity:

- First we will get an expression for the ratio of response to the excitation in terms of the **particular parameters** by giving voltage as excitation at the input port and considering the current in the output port as response (byshortcircuitingtheoutput porti.e setting V_2 aszero). i.e find out (I_2 / V_1)
- Then we will get an expression for the ratio of response to the excitation in terms of the **same parameters** bygiving voltage as excitation at the output port and considering the current in the input port as response (by shortcircuitingtheinput porti.e.setting V_1 aszero).i.efind out (I_1 / V_2)
- EquatingtheRHSofthese two expressionswouldbethecondition for reciprocity

Symmetry:

- Firstweneedtogetexpressionsrelatedtotheinputandoutputportsusingthebasic Z or Y parameter equations.
- Thentheexpressionsfor Z_{11} and Z_{22} (or Y_{11} and Y_{22}) are equatedtogetthe conmditionfor reciprocity.

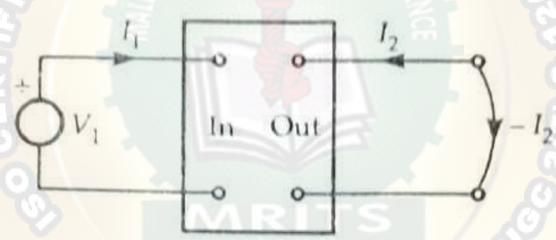
Z parameter representation: Condition for reciprocity:

Let us take a two port network with **Z** parameter defining equations as given below:

$$V_1 = Z_{11}I_1 + Z_{12}I_2 \quad V_2 =$$

$$Z_{21}I_1 + Z_{22}I_2$$

First we will get an expression for the ratio of response (I_2) to the excitation (V_1) in terms of the **Z parameters** by giving excitation at the input port and considering the current in the output port as response (by short circuiting the output port i.e. setting V_2 as zero).The corresponding **Z** parameter circuit for this condition is shown in the figure below:



(Pl note the direction of I_2 is negative since when V_2 port is shorted the current flows in the other direction)

Then the Z parameter defining equations are :

$V_1 = Z_{11}.I_1 - Z_{12}.I_2$ and 0

$= Z_{21}.I_1 - Z_{22}.I_2$

*To get the ratio of response **(I_2)** to the excitation **(V_1)** in terms of the Z parameters **I_1** is to be eliminated fom the above equations.*

So from equation 2 in the above set we will get **$I_1 = I_2 . Z_{22} / Z_{21}$**

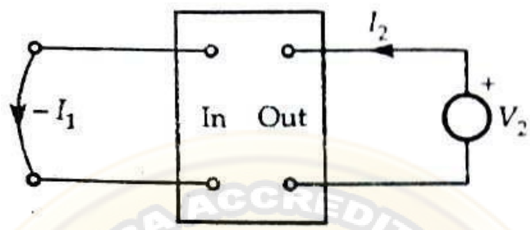
And substitute this in the first equation to get

$V_1 = (Z_{11}.I_2.Z_{22}/Z_{21}) - Z_{12}.I_2 \quad = \quad I_2[(Z_{11}.Z_{22}/Z_{21}) - Z_{12}] = I_2[(Z_{11}.Z_{22} - Z_{12}.Z_{21})/ Z_{21})] \; I_2 =$

$V_1 . Z_{21}/(Z_{11} . Z_{22} - Z_{12}.Z_{21})$

Next, we will get an expression for the ratio of response (**I_1**) to the excitation (**V_2**) in terms of the **Z**

parameters by giving excitation V_2 at the output port and considering the current I_1 in the input port as response (by short circuiting the input port i.e. setting V_1 as zero). The corresponding **Z** parameter circuit forthisconditionisshowninthefigurebelow:



(PlnotethedirectionofcurrentI_1isnegativesincewhenV_1port isshortedthecurrentflows in the other direction)

Then the Z parameter defining equations are :

$0 = -Z_{11} . I_1 + Z_{12}.I_2$ and

$V_2 = -Z_{21} . I_1 + Z_{22}. I_2$

To gettheratioofresponse(I_1) totheexcitation (V_2) in termsofthe Zparameters I_2 istobe eliminated fom the above equations.

Sofromequation1intheabovesetwewillget $I_2 = I_1. Z_{11}/ Z_{12}$

And substitute this in the second equation to get

$V_2= (Z_{22}.I_1.Z_{11}/Z_{12})-Z_{21}.I_1 = I_1[(Z_{11}.Z_{22}/ Z_{12})- Z_{21}] = I_1[(Z_{11}.Z_{22}- Z_{12}.Z_{21}) / Z_{12})]$

$I_1 = V_2 . Z_{12}/(Z_{11} . Z_{22}- Z_{12}.Z_{21})$

Assuming the input excitations V_1 and V_2 to be the same, then the condition for the out responses I_1 and I_2 to be equal would be

$$Z_{12} = Z_{21}$$

And this is the condition for the reciprocity.

Condition for symmetry:

To get this condition we need to get expressions related to the input and output ports using the basic Z parameter equations.

$$V_1=Z_{11}I_1+Z_{12}I_2 \quad V_2= Z_{21}I_1+Z_{22}I_2$$

To get the input port impedance I_2 is to be made zero. i.e V_2 should be open.

$$V_1 = Z_{11} . I_1 \text{ i.e } Z_{11} = V_1/I_1 \mid I_2=0$$

Similarly to get the output port impedance I_1 is to be made zero. i.e V_1 should be open.

$$V_2 = Z_{22} . I_2 \text{ i.e } Z_{22} = V_2/I_2 \mid I_1=0$$

Condition for Symmetry is obtained when the two port voltages are equal i.e. $V_1 = V_2$ and the two port currents are equal i.e. $I_1 = I_2$. Then

$V_1/I_1 = V_2/I_2$ i.e $Z_{11} = Z_{22}$

And hence $Z_{11} = Z_{22}$ is the condition for symmetry in Z parameters .

Y parameter representation:

Condition for reciprocity :

Let us take a two port network with **Y** parameter defining equations as given below:

$$I_1 = Y_{11}V_1 + Y_{12}V_2 \quad I_2 =$$

$$Y_{21}V_1 + Y_{22}V_2$$

First we will get an expression for the ratio of response **(I_2)** to the excitation **(V_1)** in terms of the **Y parameters** by giving excitation **(V_1)** at the input port and considering the current **(I_2)** in the output port as response(byshortcircuitingthe output porti.e.setting**V_2** aszero)

Then the second equation in **Y** parameter defining equations would become

$$I_2 = Y_{21}V_1 + 0 \text{and} \qquad I_2 / V_1 = Y_{21}$$

Then we will get an expression for the ratio of response **(I_1)** to the excitation **(V_2)** in terms of the **Y parameters** by giving excitation **(V_2)** at the output port and considering the current **(I_1)** in the input port asresponse(byshortcircuitingtheinputporti.esetting**V_1** aszero)

Then the first equation in Y parameter defining equations would become

$$I_1 = 0 + Y_{12}V_2 \quad \text{and} \quad I_1 / V_2 = Y_{12}$$

Assuming the input excitations **V_1** and **V_2** to be the same, then the condition for the out responses **I_1** and **I_2** to be equal would be

$$I_1 / V_2 = I_2 / V_1$$

AndhenceY$_{12}$= Y$_{21}$ istheconditionforthereciprocityintheTwoportnetworkwithY parameter representation.

Condition for symmetry:

To get this condition we need to get expressions related to the input and output ports (In this case Inputand output admittances) using the basic Y parameter equations

$$I_1=Y_{11}V_1+Y_{12}V_2\ I_2=$$

$$Y_{21}V_1+Y_{22}V_2$$

To get the input port admittance, V_2 is to be made zero. i.e V_2 should be shorted.

$$I_1 = Y_{11} . V_1 \quad \text{i.e} \qquad Y_{11} = I_1/V_1\ |V_2=0$$

Similarly to get the output port admittance V_1 is to be made zero. i.e V_1 should be shorted.

$$I_2 = Y_{22} . V_2 \quad \text{i.e} \qquad Y_{22} = I_2/V_2\ |\ V_1=0$$

Condition for Symmetry is obtained when the two port voltages are equal i.e. $V_1 = V_2$ and the two portcurrents are equal i.e. $I_1 = I_2$. Then

$I_1/V_1 = I_2/V_2$

And hence $Y_{11} = Y_{22}$ is thecondition for symmetryin Y parameters.

ABCD parameter representation:

Condition for reciprocity:

Let us take a two port network with ABCD parameter defining equations as given below:

$$V_1 = A.V_2 - B.I_2$$

$$I_1 = C.V_2 - D.I_2$$

First we will get an expression for the ratio of response **(I_2)** to the excitation **(V_1)** in terms of the **ABCD parameters** by giving excitation **(V_1)** at the input port and considering the current **(I_2)** in the output port as response(byshortcircuitingtheoutputporti.e.setting **V_2** aszero)

Then the first equation in the **ABCD** parameter defining equations would become

$$V_1 = 0 - B.I_2 = B.I_2$$

$$\text{i.e } I_2 / V_1 = -1/B$$

Then we will interchange the excitation and response i.e. we will get an expression for the ratio of response **(I_1)**to the excitation **(V_2)** by giving excitation **(V_2)** at the output port and considering the current **(I_1)** in the input portas response(byshortcircuitingtheinput porti.e. setting **V_1** as zero)

Then the above defining equations would become

$$0 = A.V_2 - B.I_2 \quad I_1$$

$$= C.V_2 - D.I_2$$

Substituting the value of **$I_2 = A.V_2 / B$** from first equation into the second equation we get

$$I_1 = C.V_2 - D.\ A.V_2 / B = V_2 (C - D.\ A / B)$$

$$\text{i.e} \qquad I_1/V_2 = (BC - DA) / B = -(AD - BC)/B$$

Assuming the input excitations **V_1** and **V_2** to be the same , then the condition for the out responses **I_1** and **I_2** to be equal would be

$$I_1/V_2 = I_2/V_1$$

$$\text{i.e} \qquad -(AD-BC)/B = -1/B$$

$$\text{i.e } (AD - BC) = 1$$

Andhence AD– BC= 1 isthecondition for Reciprocity intheTwoportnetwork with ABCD parameter representation.

Condition for symmetry:

Togetthiscondition weneedtoget expressions relatedtotheinput andoutput ports. Inthis caseit iseasy to use the Z parameter definitions of Z_{11} and Z_{22} for the input and output ports respectively and get their valuesintermsoftheABCDparametersasshownbelow.

$V_1 = A.V_2 - B.I_2$ I_1

$= C.V_2 - D.I_2$

$Z_{11} = V_1/I_1 \mid I_2=0$

Applying this in both the equations we get

$Z_{11} = V_1/I_1 \mid I_2=0 \quad = (A.V_2 - B.I_2)/(C.V_2 - D.I_2) \mid I_2=0$

$= (A.V_2 - B.0)/(C.V_2 - D.0)$

$= (A.V_2)/(C.V_2) = A/C$

$Z_{11} = A/C$

Similarly $Z_{22} = V_2/I_2 \mid I_1=0$

and using this in the second basic equation $I_1 = C.V_2 - D.I_2$

weget $0 = C.V_2 - D.I_2$ or $C.V_2 = D.I_2$ V_2 /

$I_2 = D/C$

$Z_{22} = D/C$

Andthe conditionforsymmetrybecomes Z_{11} = Z_{22} i.e A/C = D/C **Or A = D**

Hence A = D is the condition for Symmetry in ABCD parameter representation.

h parameter representation:

Condition for reciprocity :

Let us take a two port network with h parameter defining equations as given below:

$$V_1 = h_{11}.I_1 + h_{12}.V_2 \quad I_2$$

$$= h_{21}.I_1 + h_{22}.V_2$$

First we will get an expression for the ratio of response **(I_2)** to the excitation **(V_1)** in terms of the **h parameters** by giving excitation **(V_1)** at the input port and considering the current **(I_2)** in the output port as response(byshortcircuitingtheoutput porti.e.setting**V_2** aszero)

Then the first equation in the **h** parameter defining equations would become

$$V_1 = h_{11}.I_1 + h_{12}.0 = h_{11}.I_1$$

And in the same condition the second equation in the **h** parameter defining equations would become

$$I_2 = h_{21}.I_1 + h_{22}.0 = h_{21}.I_1$$

Dividing the second equation by the first equation we get

$$I_2/V_1 = (h_{21}.I_1)/(h_{11}.I_1) = h_{21}/h_{11}$$

Now the excitation and the response ports are interchanged and then we will get an expression for the ratio of response **(I_1)** to the excitation **(V_2)** in terms of the **h parameters** by giving excitation **(V_2)** at the output port and considering the current **(I_1)** in the input port as response (by short circuiting the input port i.e. setting **V_1** as zero)

Then the first equation in **h** parameter defining equations would become

$$0=h_{11}.I_1+h_{12}.V_2 \quad \text{i.e} \quad h_{11}.I_1 = -h_{12}.V_2$$

$$\text{i.e.} \quad I_1/V_2 = -h_{12}/h_{11}$$

Assuming the input excitations V_1 and V_2 to be the same, then the condition for the out responses I_1 and I_2 to be equal would be

$$I_1/V_2 = I_2/V_1$$

i.e = $-h_{12}/h_{11} = h_{21}/h_{11}$

$$\text{i.e.} \quad h_{12} = -h_{21}$$

And hence $[h_{12} = -h_{21}]$ is the condition for the reciprocity in the Two port network with hparameter representation.

Condition for symmetry:

To get this condition we need to get expressions related to the input and output ports. In this case also it is easy to use the Z parameter definitions of Z_{11} and Z_{22} for the input and output ports respectively and get theirvaluesintermsofthe**h** parametersasshownbelow.

h parameterequationsare: $V_1 = h_{11}.I_1 + h_{12}.V_2$

$$I_2 = h_{21}.I_1 + h_{22}.V_2$$

First let us get Z_{11} :

$$Z_{11} = V_1/I_1 \mid I_2=0$$

$$= h_{11} + h_{12}.V_2 / I_1$$

Applying the condition $I_2=0$ in the equation 2 we get

$$0 = h_{21}.I_1 + h_{22}.V_2 \text{ i.e } \quad -h_{21}.I_1 = h_{22}.V_2$$

$$\text{or } V_2 = I_1 (-h_{21} / h_{22})$$

Now substituting the value of $V_2 = I_1 (-h_{21} / h_{22})$ in the above first expression for V_1 we get

$$V_1 = h_{11}. I_1 + h_{12}. I_1.(-h_{21} / h_{22})$$

$$\text{Or } V_1 / I_1 = (h_{11}.h_{22} - h_{12}.h_{21}) / h_{22} = \Delta h / h_{22}$$

$$\text{Or } Z_{11} = \Delta h / h_{22}$$

Where $\Delta h = (h_{11}.h_{22} - h_{12}.h_{21})$ **Now let us**

get Z_{22}:

$$Z_{22} = V_2 / I_2 | I_1 = 0$$

Applying the condition $I_1 = 0$ in the second equation we get

$$I_2 = h_{21}.0 + h_{22}.V_2 \text{ i.e } V_2/I_2 = 1/h_{22}$$

$$\text{And } \quad Z_{22} = 1/h_{22}$$

Hence the condition for symmetry $Z_{11} = Z_{22}$ becomes $(\Delta h / h_{22}) = (1/h_{22})$ **i.e** $\Delta h = 1$

Hence $\Delta h = 1$ is the condition for symmetry in h parameter representation.

Table: Summary of conditions for reciprocity and symmetry for Two port networks in terms of all four parameters.

Parameter	Condition for reciprocity	Condition for symmetry
Z	$Z_{12} = Z_{21}$	$Z_{11} = Z_{22}$
Y	$Y_{12} = Y_{21}$	$Y_{11} = Y_{21}$
h	$h_{12} = -h_{21}$	$\Delta h = 1$
$ABCD$	$AD - BC = 1$	$A = D$

Different types of interconnections of two port networks:

Series Connection:

Though here only two networks are considered, the result can be generalized for any number of two port networks connected in series.

Refer the figure below where two numbers of two port networks **A** and **B** are shown connected in series. All the input and output currents & voltages with directions and polarities are shown.

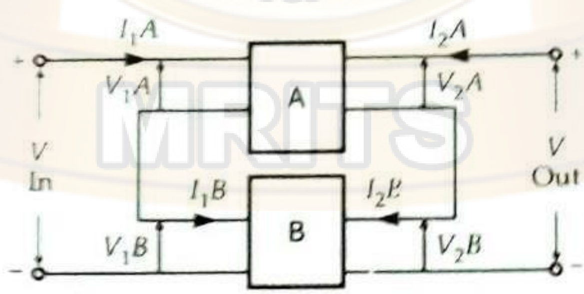


Fig : Series connection of two numbers of Two Port Networks

***Open circuit Impedance parameters* (Z)** are used in characterizing the Series connected Two port Networks. The governing equations with **Z** parameters are given below:

For network A :

$$V_{1A} = Z_{11A}\ I_{1A} + Z_{12A}\ I_{2A}$$
$$V_{2A} = Z_{21A}\ I_{1A} + Z_{22A}\ I_{2A}$$

And for network B:

$$V_{1B} = Z_{11B}\ I_{1B} + Z_{12B}\ I_{2B}$$
$$V_{2B} = Z_{21B}\ I_{1B} + Z_{22B}\ I_{2B}$$

Referring to the figure above the various voltage and current relations are:

$$I_1 \equiv I_{1A} \equiv I_{1B}$$
$$I_2 \equiv I_{2A} \equiv I_{2B}$$
$$V_2 = V_{2A} + V_{2B}$$
$$V_1 = V_{1A} + V_{1B}$$

Now substituting the above basic defining equations for the two networks into the above expressions for **V_1** and **V_2** and using the above current equalities we get:

$$V_1 = V_{1A} + V_{1B}$$
$$= (Z_{11A} I_{1A} + Z_{12A} I_{2A}) + Z_{11B} I_{1B} + Z_{12B} I_{2B}$$
$$= I_1 (Z_{11A} + Z_{11B}) + I_2 (Z_{12A} + Z_{12B})$$

And similarly

$$V_2 = V_{2A} + V_{2B}$$
$$= (Z_{21A} I_{1A} + Z_{22A} I_{2A}) + (Z_{21B} I_{1B} + Z_{22B} I_{2B})$$
$$V_2 = I_1 (Z_{21A} + Z_{21B}) + I_2 (Z_{22A} + Z_{22B})$$

Thus we get for two numbers of series connected two port networks:

$$V_1 = (Z_{11A} + Z_{11B})\, I_1 + (Z_{12A} + Z_{12B})\, I_2$$
$$V_2 = (Z_{21A} + Z_{21B})\, I_1 + (Z_{22A} + Z_{22B})\, I_2$$

Or in matrix form:

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} Z_{11A} + Z_{11B} & Z_{12A} + Z_{12B} \\ Z_{21A} + Z_{21B} & Z_{22A} + Z_{22B} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

ThusitcanbeseenthattheZparametersfortheseriesconnectedtwoportnetworksarethe sumofthe Z parameters ofthe individual twoport networks.

Cascade connection:

In this case also though here only two networks are considered, the result can be generalized for anynumber of two port networks connected in cascade.

Refer the figure below where two numbers of two port networks **X** and **Y** are shown connected in cascade.All the input and output currents & voltages with directions and polarities are shown.

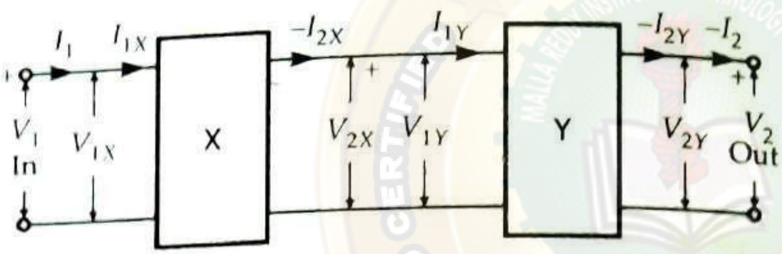


Fig 5.8: Two numbers of two port networks connected in cascade

Transmission (ABCD) parameters are easily usedin characterizing the cascade connected Two port Networks .The governing equations with transmission parameters are given below:

For network X:

$$V_{1X} = A_X V_{2X} - B_X I_{2X}$$
$$I_{1X} = C_X V_{2X} - D_X I_{2X}$$

And for network Y:

Referring to the figure above the various voltage and current relations are:

$$I_1 = I_{1X} ; -I_{2X} = I_{1Y} ; I_2 = I_{2Y}$$
$$V_1 = V_{1X} ; V_{2X} = V_{1Y} ; V_2 = V_{2Y}$$

Then the overall transmission parameters for the cascaded network in matrix form will become

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} V_{1X} \\ I_{1X} \end{bmatrix} = \begin{bmatrix} A_X & B_X \\ C_X & D_X \end{bmatrix} \begin{bmatrix} V_{2X} \\ -I_{2X} \end{bmatrix}$$

$$= \begin{bmatrix} A_X & B_X \\ C_X & D_X \end{bmatrix} \begin{bmatrix} V_{1Y} \\ I_{1Y} \end{bmatrix}$$

$$= \begin{bmatrix} A_X & B_X \\ C_X & D_X \end{bmatrix} \begin{bmatrix} A_Y & B_Y \\ C_Y & D_Y \end{bmatrix} \begin{bmatrix} V_{2Y} \\ -I_{2Y} \end{bmatrix}$$

$$= \begin{bmatrix} A_X & B_X \\ C_X & D_X \end{bmatrix} \begin{bmatrix} A_Y & B_Y \\ C_Y & D_Y \end{bmatrix} \begin{bmatrix} V_Y \\ -I_Y \end{bmatrix}$$

$$= \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_Y \\ -I_Y \end{bmatrix}$$

Where

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} A_X & B_X \\ C_X & D_X \end{bmatrix} \begin{bmatrix} A_Y & B_Y \\ D_Y & D_Y \end{bmatrix}$$

Thus it can be seen that the overall ABCD Parameter matrix of cascaded two Port Networks is theproduct ofthe ABCDmatrices ofthe individual networks.

Parallel Connection:

Though here only two networks are considered, the result can be generalized for any number of two port networks connected in parallel.

Refer the figure below where two numbers of two port networks **A** and **B** are shown connected in parallel.All the input and output currents & voltages with directions and polarities are shown.

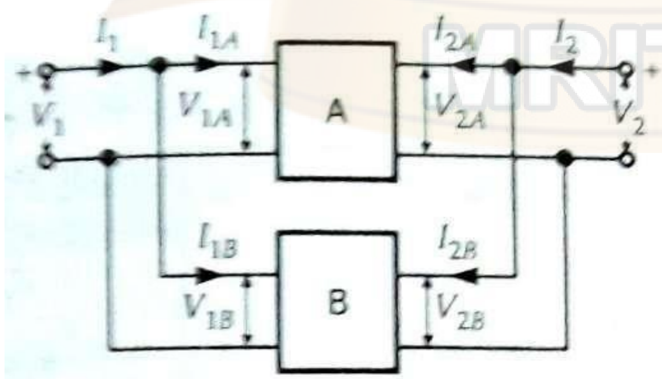


Fig 5.9: Parallel connection of two numbers of Two Port Networks

Short circuit admittance (Y) parameters are easily used in characterizing the parallel connected Twoport Networks.Thegoverningequations withYparametersaregivenbelow:

For network A:

$$I_{1A} = Y_{11A}\ V_{1A} + Y_{12A}\ V_{2A}$$
$$I_{2A} = Y_{21A}\ V_{1A} + Y_{22A}\ V_{2A}$$

And for network B:

$$I_{1B} = Y_{11B}\ V_{1B} + Y_{12B}\ V_{2B}$$
$$I_{2B} = Y_{21B}\ V_{1B} + Y_{22B}\ V_{2B}$$

Referring to the figure above the various voltage and current relations are:

$$V_1 = V_{1A} = V_{1B};\ V_2 = V_{2A} = V_{2B}$$
$$I_1 = I_{1A} + I_{1B};\ I_2 = I_{2A} + I_{2B}$$

Thus

$$\begin{aligned}
I_1 &= I_{1A} + I_{1B} \\
&= (Y_{11A}V_{1A} + Y_{12A}V_{2A}) + (Y_{11B}V_{1B} + Y_{12B}V_{2B}) \\
&= (Y_{11A} + Y_{11B})\,V_1 + (Y_{12A} + Y_{12B})\,V_2 \\
I_2 &= I_{2A} + I_{2B} \\
&= (Y_{21A}V_{1A} + Y_{22A}V_{1B}) + (Y_{21B}V_{1B} + Y_{22B}V_{2B}) \\
&= (Y_{21A} + Y_{21B})\,V_1 + (Y_{22A} + Y_{22B})\,V_2
\end{aligned}$$

Thus we finally obtain the Y parameter equations for the combined network as:

$$I_1 = (Y_{11A} + Y_{11B})\,V_1 + (Y_{12A} + Y_{12B})\,V_2$$
$$I_2 = (Y_{21A} + Y_{21B})\,V_1 + (Y_{22A} + Y_{22B})\,V_2$$

And in matrix notation it will be:

$$\begin{bmatrix} I_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} Y_{11A} + Y_{11B} & Y_{12A} + Y_{12B} \\ Y_{21A} + Y_{21B} & Y_{22A} + Y_{22B} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

Thus it can be seen thatthe overall Yparameters for theparallel connected two port networksarethe sumoftheYparametersoftheindividualtwoportnetworks.

Image impedances in terms of ABCD parameters:

Image impedances Z_{i1} and Z_{i2} of a two port network as shown in the figure below are defined as two valuesof impedances such that :

a) Whenporttwoisterminatedwith animpedance**Z_{i2}**,theinputimpedanceasseenfromPort one is **Z_{i1}** and
b) Whenportoneisterminatedwith animpedance**Z_{i1}**,theinputimpedanceasseenfromPort two is **Z_{i2}**

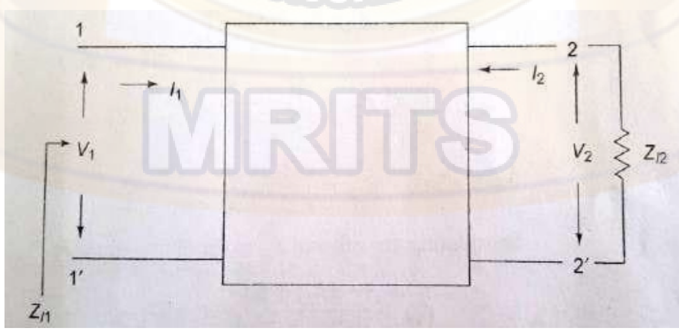


Figure 5.10: pertining to condition (a) above

Corresponding Relations are : $Z_{i1} = V_1/I_1$ and $Z_{i2} = V_2/-I_2$

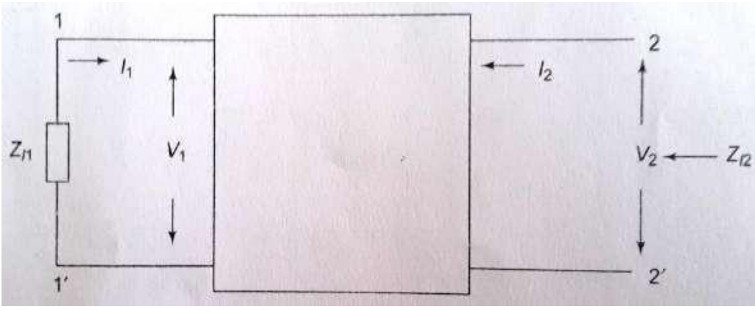


Figure 5.10: pertining to condition (b) above

Corresponding Relations are : $Z_{i1} = V_1/-I_1$ and $Z_{i2} = V_2/I_2$

Such Image impedances in terms of ABCD parameters for a two portnetwork are obtained below:

The basic defining equations for a two port network with ABCD parameters are :

$$V_1 = A.V_2 - B.I_2 \quad I_1 = C.V_2 - D.I_2$$

First let us consider condition (a).

Dividing the first equation with the second equation we get

$$Z_{i1} = \frac{V_1}{I_1} = \frac{AV_2 - BI_2}{CV_2 - DI_2}$$

Butwealsohave $\mathbf{Z_{i2}} =$ $\mathbf{V_2 / - I_2}$ andso$\mathbf{V_2 = - Z_{i2} I_2}$. SubstitutingthisvalueofV$_2$intheabove we get

$$Z_{i1} = \frac{-AZ_{i2} - B}{-CZ_{i2} - D} = \frac{AZ_{i2} + B}{CZ_{i2} + D}$$

Now let us consider the condition (b):

The basic governing equations [$V_1 = A.V_2 - B.I_2$] and [$I_1 = C.V_2 - D.I_2$] are manipulated to get

$$V_2 = \frac{DV_1}{AD - BC} - \frac{BI_1}{AD - BC}$$

$$I_2 = \frac{CV_1}{AD - BC} - \frac{AI_1}{AD - BC}$$

$$Z_{i2} = \frac{V_2}{I_2} = \frac{DV_1 - BI_1}{CV_1 - AI_1}$$

Butwealsohave $Z_{i1} =$ $V_1/-I_1$ andso$V_1 = -Z_{i1}I_1$.SubstitutingthisvalueofV_1intheabove we get:

$$Z_{i2} = \frac{DZ_{i1} + B}{CZ_{i1} + A}$$

Solving the above equations for Z_{i1} and Z_{i2} we get :

$$Z_{i1} = \sqrt{\frac{AB}{CD}}\ ; \quad Z_{i2} = \sqrt{\frac{BD}{AC}}$$

Important formulae, Equations and Relations:

- Basic Governing equations in terms of the various Parameters:
  - **Z Paramaters :** $V_1 = Z_{11}I_1 + Z_{12}I_2$

    $V_2 = Z_{21}I_1 + Z_{22}I_2$
  - **Y Parameters:** $I_1 = Y_{11}V_1 + Y_{12}V_2$

    $I_2 = Y_{21}V_1 + Y_{22}V_2$
  - **h Parameters:** $V_1 = h_{11}.I_1 + h_{12}.V_2$

    $I_2 = h_{21}.I_1 + h_{22}.V_2$

ABCD Parameters: $V_1 = A.V_2 - B.I_2$

$I_1 = C.V_2 – D.I_2$

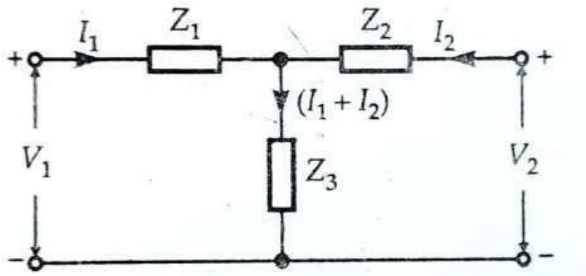
- Conditions for Reciprocity and symmetry for Two Port Networks in terms of the variousparameters :

Parameter	Condition for reciprocity	Condition for symmetry
Z	$Z_{12} = Z_{21}$	$Z_{11} = Z_{22}$
Y	$Y_{12} = Y_{21}$	$Y_{11} = Y_{21}$
h	$h_{12} = -h_{21}$	$\Delta h = 1$
ABCD	$AD - BC = 1$	$A = D$

- **Relations of Interconnected two port Networks :**
- *The overall **Z parameters** for the **series** connected two port networks are the **sum of the Z parameters of the individual two port networks.***
- *The overall **Y parameters** for the **parallel** connected two port networks are the **sum of the Y parameters of the individual two port networks**.*
- *The overall **ABCD Parameter matrix** of **cascaded** two Port Networks is the **product of the ABCD matrices of the individual networks.***

Illustrative problems :

Example 1: Find the Z Parameters of the following Two Port Network and draw it's equivalent circuit in terms of Z_1 Z_2 and Z_3 .



Solution: Applying KVL to the above circuit in the two loops ,with the current notation as shown, the loop equations for V_1 and V_2 can be written as:

$$V_1 = I_1 Z_1 + (I_1 + I_2) Z_3$$

or $$V_1 = (Z_1 + Z_3) I_1 + Z_3 I_2 \quad \text{...(i)}$$

and $$V_2 = I_2 Z_2 + (I_2 + I_1) Z_3$$

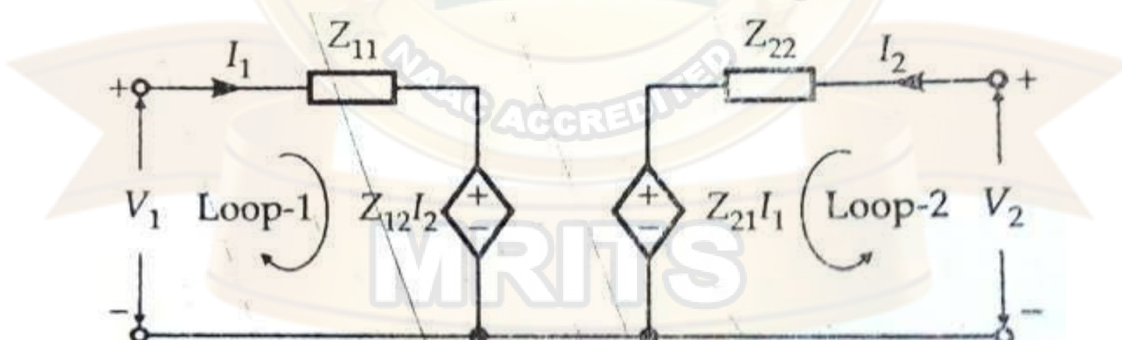
or $$V_2 = Z_3 I_1 + (Z_2 + Z_3) I_2 \quad \text{...(ii)}$$

Comparing the equations (i) and (ii) above with the standard expressions for the Z parameter equationswe get:

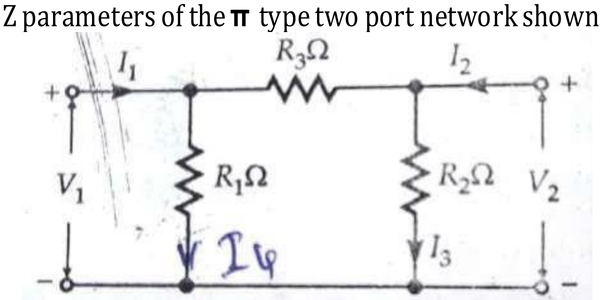
$$Z_{11} = Z_1 + Z_3;\ Z_{12} = Z_3;$$

$$Z_{21} = Z_3\ ;\ Z_{22} = Z_2 + Z_3$$

Equivalent circuit in terms of Z_1 Z_2 and Z_3 is shown below.



Example 2: Determine the Z parameters of the **π** type two port network shown in the figure below.



Solution:

From the basic Z parameter equations We know that

$Z_{11} = V_1/I_1 \mid I_2=0$ Z_{12}

$= V_1/I_2 \mid I_1=0$ $Z_{21} =$

$V_2/I_1 \mid I_2=0$ $Z_{22} =$

$V_2/I_2 \mid I_1=0$

We will first find out Z_{11} and Z_{21} which are given by the common condition $I_2 = 0$

1. We can observe that $Z_{11} = V_1/I_1$ with $I_2=0$ is the parallel combination of R_1 and $(R_2 + R_3)$.

$$\therefore \quad Z_{11} = R_1 (R_2 + R_3) / (R_1+R_2 + R_3)$$

2. **$Z_{21} = V_2/I_1 \mid I_2=0$**

By observing the network we find that the current I_1 is dividing into I_3 and I_4 as shown in the figure where I_3 is flowing through R2 (and R_3 also since $I_2=0$)

Hence $V_2 = I_3 \, x R_2$

From the principle of current division we find that $I_3 = I_1 . R_1 / (R_1+R_2 + R_3)$ Hence

$V_2 = I_3 \, x R_2 = [I_1.R_1 / (R_1+R_2 + R_3)].R_2 = I_1.R_1 R_2/(R_1+R_2+R_3)$ And

$V_2/I_1 = R_1 R_2 / (R_1+R_2 + R_3)$

$$\therefore \quad Z_{21} = R_1 R_2 / (R_1+R_2 + R_3)$$

Next we will find out Z_{12} and Z_{22} which are given by the common condition $I_1 = 0$ 3. **$Z_{12} =$**

$V_1/I2 \mid I_1=0$

By observing the network we find that the current I_2 is now dividing into I_3 and I_4 as shown in the figure where I_4 is flowing through R_1 (and R_3 also since $I_1 = 0$)

Hence $V_1 = I_4 \times R_1$

Again from the principle of current division we find that $I_4 = I_2 . R_2 / (R_1+R_2+R_3)$ Hence V_1

$= I_4 \times R_1 = [I_2.R_2/(R_1+R_2+R_3)].R_1 = I_2.R_1 R_2/(R_1+R_2+R_3)$ And

$$V_1/I_2 = R_1 R_2 / (R_1+R_2+R_3)$$

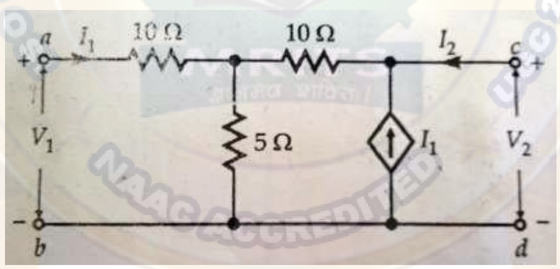
$$\therefore \quad Z_{12} = R_1 R_2 /(R_1+R_2+R_3)$$

4. We can again observe that $Z_{22} = V_2/I_2$ with $I_1=0$ is the parallel combination of R_2 and $(R_1 + R_3)$

.

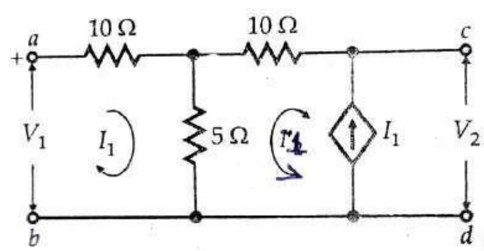
$$\therefore \quad Z_{22} = R_2 (R_1+R_3) /(R_1+R_2+R_3)$$

Example 3 : Determine the Z parameters of the network shown in the figure below.



1). We will first find out Z_{11} and Z_{21} which are given by the common condition $I_2 = 0$ (Output open circuited)

With this condition the circuit is redrawn as shown below.



Since the current source is there in the second loop which is equal to I_1 and I_2 is zero, only current I_1 flows through the right hand side resistance of 10Ω and both currents I_1(both loop currents) passthrough theresistanceof5Ωasshownintheredrawnfigure.

Now the equation for loop one is given by :

$V_1 = 10x\ I_1 + 5\ (2\ I_1) = 20\ I_1$ and $V_1/I_1 = 20\Omega$

$$\therefore \mathbf{V_1/I_1}\ \Big|\ \mathbf{I_2}=0 \quad = \quad \mathbf{Z_{11}} = 20\Omega$$

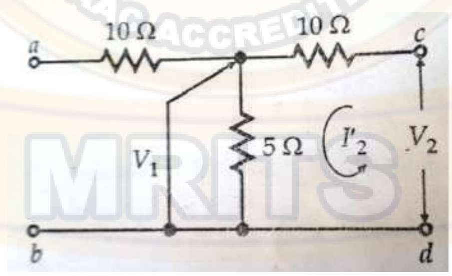
Next the equation for loop two is given by :

$V_2 = 10x\ I_1 + 5\ (2\ I_1) = 20\ I_1$ and $V_2/I_1 = 20\Omega$

$$\therefore \mathbf{V_2/I_1}\ \Big|\ \mathbf{I_2}=0 \quad = \quad \mathbf{Z_{21}} = 20\Omega$$

2). Next we will find out Z_{12} and Z_{22} which aregiven by thecommon condition $I_1 = 0$ (input open circuited)

With this condition the circuit is redrawn as shown below.



Now since the current I_1 is zero ,the current source of I_1 would no longer be there in the output loop and it is removed as shown in the redrawn figure. Further since input current I_1= 0 ,there would be no current in the input side 10Ω and the same current I_2 only flows through common resistance of 5 Ω and output side resistance of 10 Ω .With these conditions incorporated, now we shall rewrite the two loop equations (for inputV_1andoutputV_2)toget***Z_{12} and Z_{22}***

Equation for loop one is given by :

$V_1 = 5 I_2$ and $V_1/I_2 = 5\Omega$

$$\therefore \mathbf{V}_1/\mathbf{I}_2 \,|\, \mathbf{I_1}=0 \quad = \quad \mathbf{Z}_{12} = 5\Omega$$

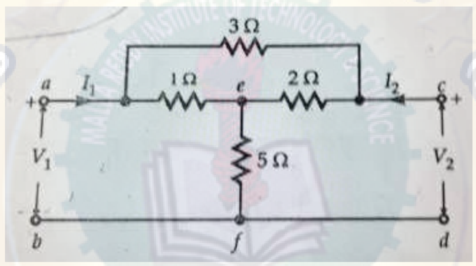
And the equation for loop two is given by:

$V_2 = 10 \times I_2 + 5 \times I_2 = 15 I_2$ and $V_2/I_2 = 15\Omega$

$$\therefore V_2/I_2 \Big|_{I_1=0} = Z_{22} = 15\Omega$$

Finally: $Z_{11} = 20\Omega$; $Z_{12} = 5\Omega$; $Z_{21} = 20\Omega$; $Z_{22} = 15\Omega$

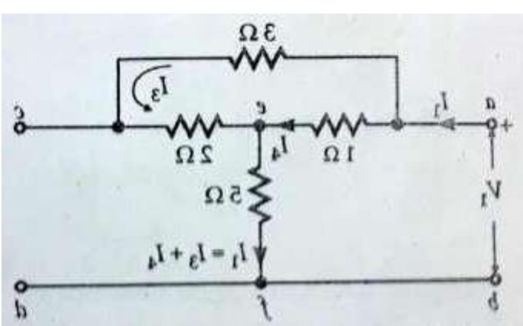
Example 4: Obtainthe open circuit parameters ofthe Bridged T network shown inthefigure below.



Open circuit parameters are same as Z parameters.

1). Wewill first find out Z_{11} and Z_{21} which aregiven by thecommon condition $I_2 = 0$ (Output open circuited)

With this condition the circuit is redrawn as shown below.



From the inspection of the figure in this condition it can be seen that (since I_2 is zero) the two resistances i.e the bridged arm of 3Ω and output side resistance of 2Ω are in series and together are in parallelwith the input side resistance of 1Ω.

Hence the loop equation for V_1 can be written as:

$V_1 = I_1 \times [(3+2) \parallel 1 + 5] = I_1 \times 35/6$ and $V_1/I_1 = 35/6$

$$\therefore\ V_1/I_1 \big|_{I_2=0} \quad = \quad Z_{11} = 35/6\Omega$$

Next the loop equation for V_2 can be written as :

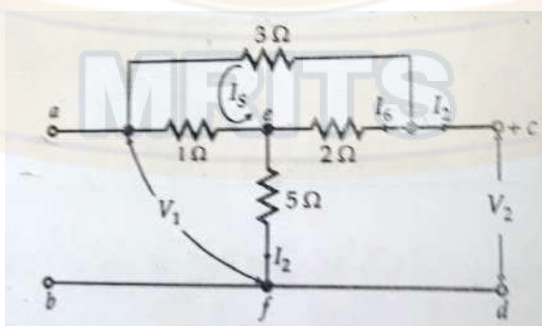
$V_2 = I_3 \times 2 + I_1 \times 5$

But we know from the principle of current division that the current $I_3 = I_1 \times [1/(1+2+3)] = I_1 \times 1/6$ Hence $V_2 = I_1 \times 1/6 \times 2 + I_1 \times 5 = I_1 \times 16/3$ and $V_2 / I_1 = 16/3\,\Omega$

$$\therefore\ V_2/I_1 \big|_{I_2=0} \quad = \quad Z_{21} = 16/3\ \Omega$$

2). Next we will find out Z_{12} and Z_{22} which aregiven by thecommon condition $I_1 = 0$ (input open circuited)

With this condition the circuit is redrawn as shown below.



From the inspection of the figure in this condition it can be seen that (since I_1 is zero) the two resistances i.e the bridged arm of 3Ω and input side resistance of 1Ω are in series and together are in parallel with the output side resistance of 2Ω. Further $I_2 = I_5 + I_6$

Hence the loop equation for V_1 can be written as : $V_1 = I_5$

$x1 + I_2 x5$

But we know from the principle of current division that the current $I_5 = I_2 x[2/(1+2+3)] = I_2 x1/3$ Hence $V_1 = I_2 x1/3$

$x1 + I_2 x5 = I_2 x16/3$ and $V_1 / I_2 = 16/3\ \Omega$

$$\therefore\ V_1/I_2\ \big|_{I_1=0} = Z_{12} = 16/3\ \Omega$$

Next the loop equation for V_2 can be written as:

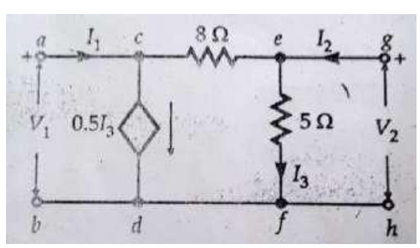
$V_2 = I_6\ x2 + I_2 x5$

But we know from the principle of current division that the current $I_6 = I_2 x[1/(1+2+3)] = I_2 x (3+1)/6 = (I_2 x 2/3)$

Hence $V_2 = I_2 x (2/3)x 2 + I_2 x5 = I_2 x 19/3$ and $V_2/I_2 = 19/3$

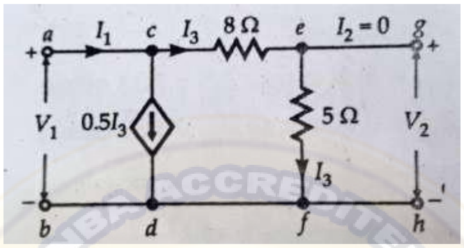
$$\therefore\ V_2/I_2\ \big|_{I_2=0} = Z_{22} = 19/3\ \Omega$$

Example 5 : Obtain Z parameters of the following **π** network with a controlled current source of 0.5 I_3 in the input port.



1). We will first find out Z_{11} and Z_{21} which are given by the common condition $I_2 = 0$ (Output open circuited)

With this condition the circuit is redrawn as shown below.



In this condition we shall first apply Kirchhoff's current law to the node **'c'**:

Then $I_1 = 0.5I_3 + I_3$ (I_3 being the current through the resistances of 8 Ω and 5 Ω) i.e $I_1 =$

$0.5I_3 + I_3$ or $I_1 = 1.5I_3$ or $I_3 = I_1/1.5$ i.e $I_3 = (2/3)I_1$

Now we also observe that $V_1 = I_3(8+5) = 13.\ I_3$

Using the value of $I_3 = (2/3)I_1$ into the above expression we get $V_1 =$

$13(2/3)I_1$ and $V_1/\ I_1 = 26/3 = 8.67$

$$\therefore\ V_1/I_1\ \big|_{I_2=0} = Z_{11} = 8.67\Omega$$

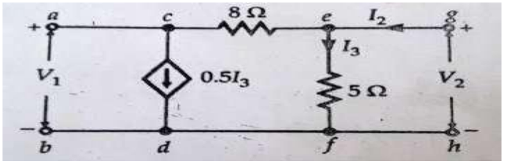
Next we also observe that $V_2 = 5.\ I_3$ and substituting the above value of $I_3 = (2/3)I_1$ into this expression for V_2 we get:

$V_2 = 5\ .\ I_3$ i.e $V_2 = 5\ .\ (2/3)I_1$ i.e $V_2\ /\ I_1 = 10/3 = 3.33\Omega$

$$\therefore\ V_2/I_1\ \big|_{I_2=0} = Z_{21} = 3.33\ \Omega$$

2). Next we will find out Z_{12} and Z_{22} which are given by the common condition $I_1 = 0$ (input open circuited)

With this condition the circuit is redrawn as shown below.



In this condition now we shall first apply Kirchhoff's current law to the node **'e'**:

Then $I_2 = 0.5I_3 + I_3$ ($0.5.I_3$ being the current through the resistance of 8 Ω and I_3 being the current through the resistances of 5 Ω)

i.e $I_2 = 0.5I_3 + I_3$ or $I_2 = 1.5I_3$ or $I_3 = I_2/1.5$ i.e $I_3 = (2/3)I_2$

Now we also observe that $V_1 = (-0.5I_3 \times 8 + I_3 \times 5) = I_3$ (it is to be noted here carefully that – sign is to be taken before $0.5I_3 \times 8$ since the current flows through the resistance of 8 Ω now in the reverse direction.

Using the value of $I_3 = (2/3)I_2$ into the above expression for V_1 we get $V_1 =$

$(2/3)I_2$ and $V_1/I_2 =$ 0.67

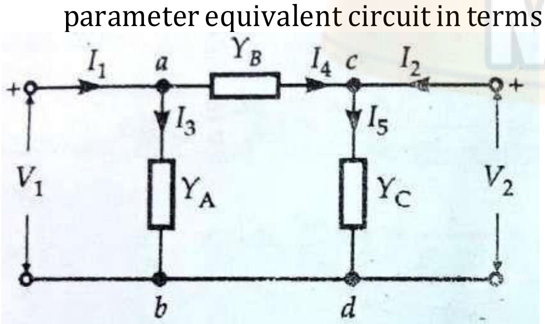
$$\therefore\ V_1/I_2 \,\big|_{I_1=0} = Z_{12} = 0.67\Omega$$

Next we also observe that $V_2 = 5.I_3$ and substituting the above value of $I_3 = (2/3)I_2$ into this expression for V_2 we get :

$V_2 = 5 . I_3$ i.e $V_2 = 5 . (2/3)I_2$ i.e $V_2 / I_2 = 10/3 = 3.33\Omega$

$$\therefore\ V_2/I_2 \,\big|_{I_1=0} = Z_{21} = 3.33\ \Omega$$

Example 6 : Find the Y parameters of the following **π** type two port network and draw it's Y parameter equivalent circuit in terms of the given circuit parameters.



Applying KCL at node (a) we get

$$I_1 = I_3 + I_4$$
$$I_1 = V_1 Y_A + (V_1 - V_2) Y_B$$
$$I_1 = V_1 (Y_A + Y_B) + (-Y_B) V_2 \quad \text{--(i)}$$

Similarly applying KCL to node (c) we get

$$I_2 = I_5 - I_4$$
$$I_2 = V_2 Y_C - (V_1 - V_2) Y_B$$
$$I_2 = (-Y_B) V_1 + (Y_C + Y_B) V_2 \quad \text{...(ii)}$$

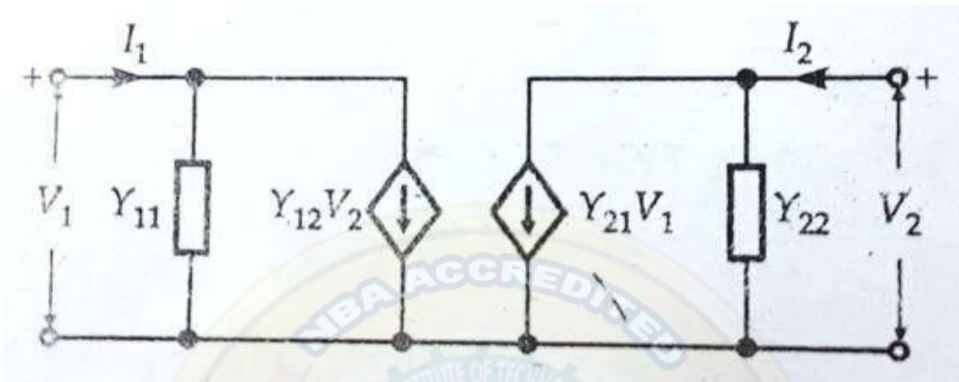
Comparing the equations (i) and (ii) above with the standard expressions for the Y parameter equationswe get:

$$Y_{11} = (Y_A + Y_B)\,;\; Y_{12} = -Y_B$$
$$Y_{21} = -Y_B\,;\; Y_{22} = Y_C + Y_B.$$

Observing the equations (i) and (ii) above we find that :

- ThetermsV$_1$(Y$_A$+Y$_B$)and V$_2$(Y$_c$+Y$_B$)arethecurrents throughthe admittances Y_{11} andY_{22} and
- Theterms -Y$_B$.V$_2$ and -Y$_B$.V$_1$ arethedependentcurrentsourcesintheinputandtheoutput ports respectively.

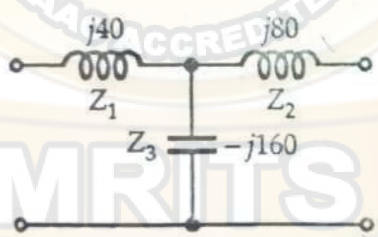
These observations are reflected in the equivalent circuit shown below.



In the above figure $Y_{11} = (Y_A + Y_B)$ & $Y_{22} = (Y_c + Y_B)$ are the admittances and

$Y_{12} .V_2 = -Y_B .V_2$ & $Y_{21} .V_1 = -Y_B .V_1$ are the dependent current sources

Example 7: Find the Y parameters of the following network



Solution: We will solve this problem in two steps.

1. We shall first express the Z parameters of the given T network in terms of the impedances Z_1, Z_2 and Z_3 using thestandardformulaswealreadyknowandsubstitutethegivenvaluesof Z_1, Z_2 and Z_3.

$$Z_{11} = Z_1 + Z_3 = -j\,120\,;$$
$$Z_{12} = Z_3 = -j\,160$$
$$Z_{21} = Z_3 = -j\,160\,;$$
$$Z_{22} = Z_2 + Z_3 = -j\,80$$

2. Then convert the values of the Z parameters into Y parameters i.e express the Y parameters in terms of Z parameters using again the standard relationships.

$$Y_{11} = \frac{Z_{22}}{Z_{11}Z_{22} - Z_{12}Z_{21}}$$

$$= \frac{-j\,80}{(-j\,120)(-j\,80) - (-j\,160)^2}$$

$$= \frac{-j\,80}{16{,}000} = \frac{-j}{\mathbf{200}}\ \textbf{mho.}$$

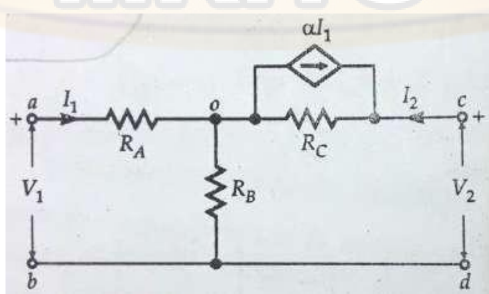
$$Y_{12} = Y_{21} = \frac{-Z_{12}}{Z_{11}Z_{22} - Z_{12}Z_{21}}$$

$$= \frac{j\,160}{16{,}000} = \frac{j}{\mathbf{100}}\ \textbf{mho.}$$

$$Y_{22} = \frac{Z_{11}}{Z_{11}Z_{22} - Z_{12}Z_{21}}$$

$$= \frac{-j\,120}{16{,}000} = \frac{-j}{\mathbf{133.33}}\ \textbf{mho.}$$

Example 8: Find the **' h'** parameters of the network shown below. (fig12.34)



First let us write down the basic **' h'** parameter equations and give the definitions of the **' h'** parameters.

$$V_1 = h_{11}.I_1 + h_{12}.V_2 \quad I_2$$

$$= h_{21}.I_1 + h_{22}.V_2$$

$h_{11} = V_1/I_1$ with $V_2 = 0$ **$h_{21} = I_2/I_1$ with $V_2 = 0$**

$h_{12} = V_1/V_2$ with $I_1 = 0$ **$h_{22} = I_2/V_2$ with $I_1 = 0$**

Now

1). We will first find out h_{11} and h_{21} which are given by the common condition $V_2 = 0$ (Output short circuited)

In this condition it can be observed that the resistance R_C and the current source αI_1 become parallel with resistance R_B.

For convenience let us introduce a temporary variable **V** as the voltage at the node **'o'**. Then the current through the parallel combination of R_B and R_C would be equal to

$$\frac{V}{\frac{R_B R_C}{R_B + R_C}} = \frac{V(R_B + R_C)}{R_B R_C}$$

Then applying KCL at the node **'o'** we get

$$I_1 = \frac{V(R_B + R_C)}{R_B R_C} + \alpha I_1$$

$$I_1(1-\alpha) = \frac{V(R_B + R_C)}{R_B R_C}$$

$$\therefore \quad V = \frac{(1-\alpha) I_1 R_B R_C}{(R_B + R_C)}$$

Next applying KVL at input port we get $V_1 = I_1.R_A + V$ and $V_1/I_1 = R_A + V/I_1$ Now using the value of V we obtained above in this expression for V_1/I_1 we get

$$h_{11} = \frac{V_1}{I_1} = R_A + \frac{(1-\alpha)R_B R_C}{R_B + R_C}$$
$$= \frac{R_A(R_B + R_C) + (1-\alpha)R_B R_C}{R_B + R_C} \text{ ohm.}$$

Again from inspection of the figure above it is evident that

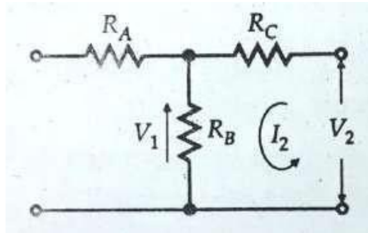
$$I_2 = -\left(\alpha I_1 + \frac{V}{R_C}\right)$$
$$I_2 = -\alpha I_1 - \frac{(1-\alpha) I_1 R_B}{(R_B + R_C)}$$

Therefore

$$h_{21} = \left.\frac{I_2}{I_1}\right|_{V_2=0} = -\alpha - \frac{(1-\alpha) R_B}{(R_B + R_C)}$$
$$= -\frac{(\alpha R_C + R_B)}{(R_B + R_C)}.$$

2). Next we will find out h_{12} and h_{22} which are given by the common condition $I_1 = 0$ (Input open circuited)

Now since I_1 is zero , the current source disappears and the circuit becomes simpler as shown in the figure below.



Now applying KVL at the output port we get:

$$V_2 = I_2 (R_B + R_C)$$

$$\left.\frac{I_2}{V_2}\right|_{I_1=0} = h_{22} = \left(\frac{1}{R_B + R_C}\right) \text{mho.}$$

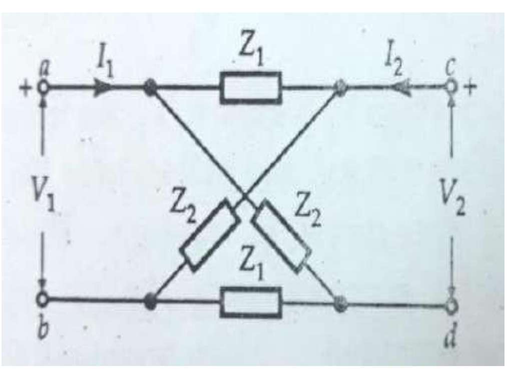
Again under thiscondition:

$$V_1 = I_2 R_B$$

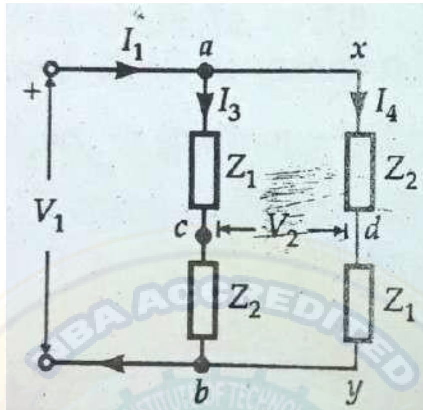
$$\therefore \quad h_{12} = \left.\frac{V_1}{V_2}\right|_{I_1=0} = \frac{I_2 R_B}{I_2 (R_B + R_C)}$$

$$= \frac{R_B}{R_B + R_C}.$$

Example 9 : Z parameters of the lattice network shown in the figure below.



First we shall redraw the given lattice network in a simpler form for easy analysis as shown below.



We will then find out Z_{11} and Z_{21} which are given by the common condition $I_2=0$ (Output open circuited)

It can be observed that the impedances in the two arms **'ab'** and **'xy'** are same i.e $Z_1 + Z_2$ and their parallel combination is $(Z_1 + Z_2)/2$

Hence applying KVL at the input port we get

$$V_1 = I_1\left(\frac{Z_1 + Z_2}{2}\right)$$

$$\frac{V_1}{I_1} = Z_{11}\big|_{I_2=0} = \frac{Z_1 + Z_2}{2}$$

Next we find that

$$V_2 = V_c - V_d = (V_1 - I_3 Z_1) - (V_1 - I_4 Z_2)$$
$$= I_4 Z_2 - I_3 Z_1$$

(V_C and V_D being the potentials at points '**c'** and '**d')**

It can also be observed from the simplified circuit that the currents I_3 and I_4 through the branches **'ab'** and **'xy'** are equal since the branch impedances are same and same voltage V_1 is applied across both the branches. Hence the current I divides equally as I_3 and I_4

i.e $I_3 = I_4 = I/2$

Now substituting these values of I_3 and I_4 in the expression for V_2 above:

$$V_2 = \frac{I_1}{2} \times Z_2 - \frac{I_1}{2} \times Z_1 = \frac{Z_2 - Z_1}{2} . I_1$$

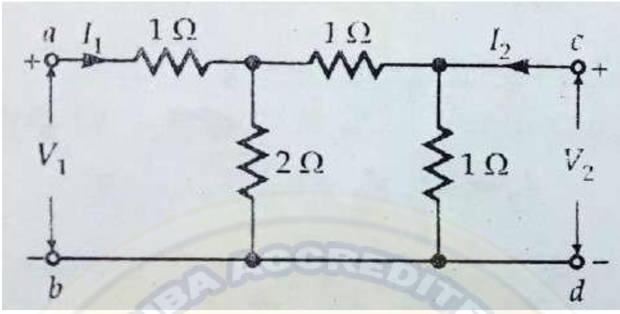
$$\frac{V_2}{I_1} = Z_{21}\Big|_{I_2=0} = \frac{Z_2 - Z_1}{2}$$

As can be seen the circuit is both symmetrical and Reciprocal and hence :

$$Z_{11} = Z_{22} = \frac{Z_1 + Z_2}{2}$$

$$Z_{12} = Z_{21} = \frac{Z_2 - Z_1}{2}.$$

Example 10: Find the transmission parameters of the following network (fig 12.51)



First let us write down the basic ABCD parameter equations and give their definitions.

$V_1 = A.V_2 - B.I_2$ I_1

$= C.V_2 - D.I_2$

A = V_1/V_2 with I_2 = 0

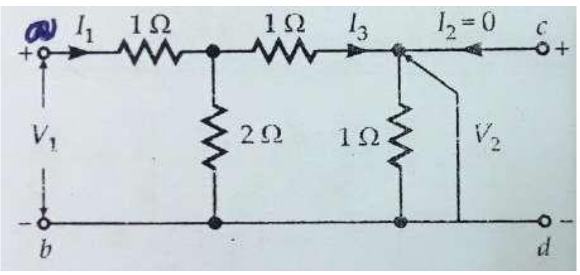
C = I_1/V_2 with I_2 = 0

B = $V_1/-I_2$ with V_2 = 0

D = $I_1/-I_2$ with V_2 = 0

1).WewillthenfindoutAandCwhicharegivenbythecommonconditionI$_2$=0(Outputopen circuited)

The resulting circuit in this condition is redrawn below.



Applying KVL we can write down the two mesh equations and get the values of A and C :

$$V_1 = I_1 \times 1 + (I_1 - I_3)2$$

or $$V_1 = 3I_1 - 2I_3 \quad \text{...(i)}$$

and $$0 = (I_3 - I_1)2 + I_3(1+1) = 4I_3 - 2I_1$$

$$\therefore \quad I_3 = \frac{1}{2} I_1 \quad \text{...(ii)}$$

Utilising (ii) in (i),

$$V_1 = 3I_1 - 2 \times \frac{1}{2} I_1 = 2I_1 \quad \text{...(iii)}$$

∴ Again,

$$V_2 = I_3 \times 1 = \frac{1}{2} I_1 \quad \text{...(iv)}$$

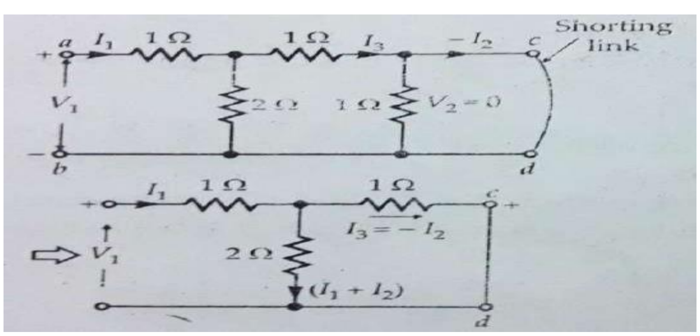
$$\therefore \quad \left.\frac{I_1}{V_2}\right|_{I_2=0} = 2 \text{ mho} = C.$$

Dividing equation (iii) by (iv),

$$\left.\frac{V_1}{V_2}\right|_{I_2=0} = 4 = A.$$

2.) Next we will find out Band Dwhich aregiven by thecommon condition $V_2 = 0$ (Output short circuited)

The resulting simplified network in this condition is redrawn below.



The voltage at the input port is given by : $V_1 = I_1 x1 + (I_1 + I_2) x2$

i.e. $V_1 = 3I_1 + 2I_2$ .... (i)

And the mesh equation for the closed mesh through 'cd' is given by : $0 = I_2 x1 + (I_1 + I_2) x2$ or $3 I_2 + 2 I_1 = 0$ or

$I_1 = -(3/2). I_2$ .... (ii)

Using equation (ii) in the equation (i) above we get :

$V_1 = -(9/2) I_2 + 2I_2 = -(5/2)I_2$

Or $V_1 / -I_2 = B = (5/2)$

And from equation (ii) above we can directly get

$I_1 / -I_2 = D = 3/2$

Hence the transmission parameters can be written in matrix notation as :

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 4 & \frac{5}{2} \\ 2 & \frac{3}{2} \end{bmatrix}$$

Here we can see that AD – BC = 1 and A ≠ D

Hence the network is Symmetrical but not Reciprocal.

UNIT-III:

Locus diagrams:

- Resonance and Magnetic Circuits:
- Locus diagrams – Series and Parallel RL, RC, RLC circuits with variation ofvarious parameters –
- Resonance-Series and Parallel circuits,
- Concept of band width and quality factor.
- Magnetic Circuits- Faraday's laws of electromagnetic induction,
- Concept of self and mutual inductance,
- Dot convention, Coefficient of coupling,
- Composite magnetic circuits,
- Analysis of series and parallel magnetic circuits.

Locus Diagrams with variation of various parameters:

Introduction: In AC electrical circuits the magnitude and phase of the current vector depends upon the values of R,L&C when the applied voltage and frequency are kept constant. The path traced by the terminus (tip) of the current vector when the parameters R,L&C are varied is called the current ***Locus diagram*** . Locus diagrams are useful in studying and understanding the behavior of the RLC circuits when one of these parameters is varied keeping voltage and frequency constant.

In this unit, Locus diagrams are developed and explained for series RC,RL circuits and Parallel LC circuits along with their internal resistanceswhentheparametersR,LandCarevaried.

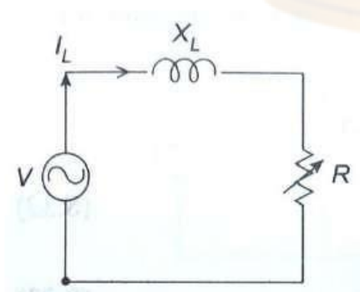
The term circle diagram identifies locus plots that are either circular or semicircular. The defining equations of such circle diagramsarealsoderivedinthisunitforseriesRCandRL diagrams.

In both series RC,RL circuits and parallel LC circuits resistances are taken to be in series with L and C to highlight the fact that all practical L and C components will have at least a small value of internal resistance. Series RL circuit with varying Resistance R:

Refer to the series RL circuit shown in the figure (a) below with constant X_L and varying R. The current I_L lags behind the applied voltage V by a phase angle $\Theta = \tan^{-1}(X_L/R)$ for a given value of R as shown in the figure (b) below. When R=0 we can see that the current is maximum equal to V/X_L and lies along the I axis with phase angle equal to 90^0. When R is increased from zero to infinity the current gradually reduces from V/X_L to 0 and phase angle also reduces from 90^0 to 0°

As can be seen from the figure, the tip of the current vector traces the path of a semicircle

With its diameter along the +ve I axis.



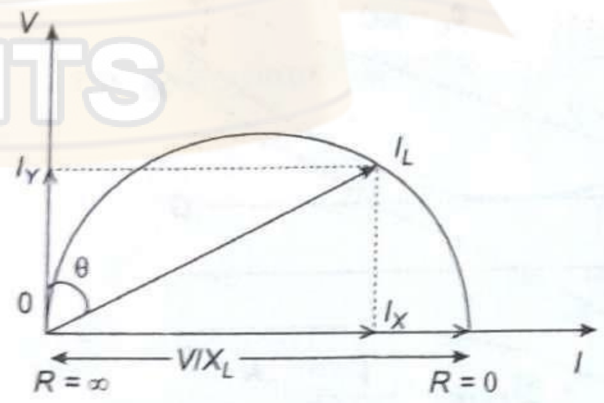


Fig 4.1(a): Series RL circuit with Varying Resistance R

Fig 4.1(b): Locus of current vector I_L with variation of R

The related equations are:

$I_L = V/Z$ $\quad$ $\sin\Theta = X_L/Z$ or $Z = X_L/\sin\Theta$ and $\cos\Theta = R/Z$

Therefore $I_L = (V/X_L)\sin\Theta$

For constant V and X_L the above expression for I_L is the polar equation of a circle with diameter (V/X_L) as shown in the figure above.

Circle equation for the RL circuit: (with fixed reactance and variable Resistance):

The X and Y coordinates of the current I_L are $I_X = I_L$ $\sin\Theta$ $\quad$ $I_Y = I_L \cos\Theta$

From the relations given above and earlier we get

$$I_X = (V/Z)(X_L/Z) = VX_L/Z^2 \quad \text{--------} \quad (1)$$

and

$$I_Y = (V/Z)(R/Z) = VR/Z^2 \quad \text{--------} \quad (2)$$

Squaring and adding the above two equations we get

$$I_X^2+I_Y^2 = V^2(X_L^2+R^2)/Z^4 = (V^2Z^2)/Z^4 = V^2/Z^2 \quad \text{--------} \quad (3)$$

From equation (1) above we have $Z^2 = VX_L/I_X$ and substituting this in the above equation (3) we get:

$$I_X^2+I_Y^2 = V^2/(VX_L/I_X) = (V/X_L)I_X \quad \text{or}$$

$$I_X^2+I_Y^2-(V/X_L)I_X = 0$$

Adding $(V/2X_L)^2$ to both sides ,the above equation can be written as

$$[I_X-V/2X_L]^2+I_Y^2 = (V/2X_L)^2 \quad \text{--------} \quad (4)$$

Equation (4) above represents a circle with a radius of $(V/2X_L)$ and with it's coordinates of the centre as $(V/2X_L, 0)$

Series RC circuit with varying Resistance R:

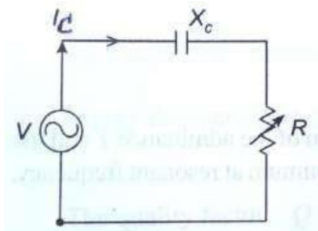
Refer to the series RC circuit shown in the figure (a) below with constant X_C and varying R. The current I_C leads the applied voltage V by a phase angle $\Theta = \tan^{-1}(X_C/R)$ for a given value of R as shown in the figure (b) below.WhenR=0 wecanseethatthecurrentismaximumequalto− V/X_C and liesalongthe negative Iaxis with phase angle equalto− 90^0. When R is increased from zero to infinity the current gradually reduces from $-V/X_C$ to 0 and phase angle also reduces from -90^0 to 0^0. As can be seen from the figure, the tip of the current vector traces the path of a semicircle butnowwithitsdiameteralongthenegative Iaxis.

Circle equation for the RC circuit: (with fixed reactance and variable Resistance):

Inthesameway aswegotforthe SeriesRLcircuitwithvaryingresistancewecangetthecircle equation for an RC circuit with varyingresistance as:

$$[I_X + V/2X_C]^2 + I_Y^2 = (V/2X_C)^2$$

whose coordinates of the centre are $(-V/2X_C, 0)$ and radius equal to $V/2X_C$



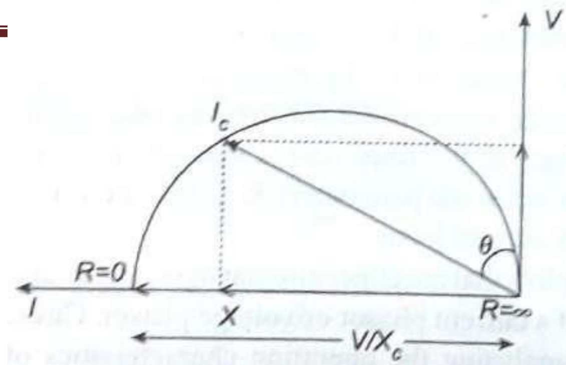
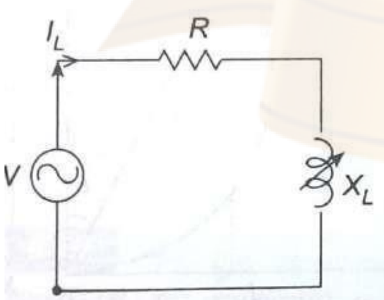


Fig4.2(a): Series RCcircuit with VaryingResistanceR

Fig4.2(b):LocusofcurrentvectorI$_C$ with variation ofR

Series RL circuit with varying Reactance X_L:

Refer to the series RL circuit shown in the figure (a) below with constant R and varying X_L. The current I_L lags behind the applied voltage V by a phase angle $\theta = \tan^{-1}(X_L/R)$ for a given value of R as shown in the figure (b) below. When $X_L = 0$ we can see that the current is maximum equal to V/R and lies along the +ve V axis with phase angle equal to 0^0. When X_L is increased from zero to infinity the current gradually reduces from V/R to 0 and phase angle increases from 0^0 to 90^0. As can be seen from the figure, the tip of the current vector traces the path of a semicircle with its diameter along the +ve V axis and on to its rightside.



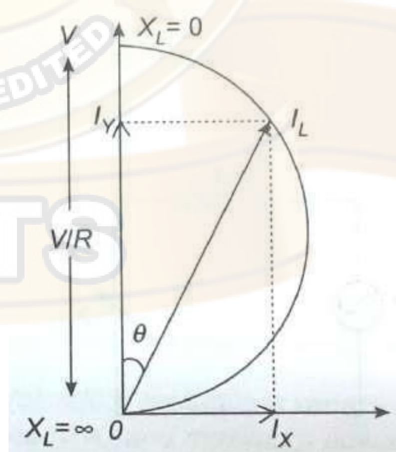
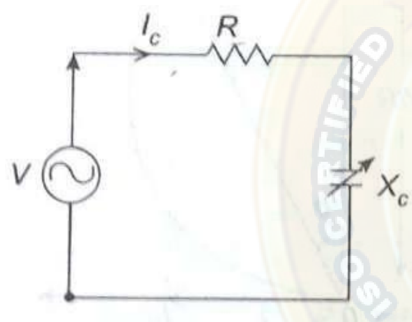


Fig 4.3(a): Series RL circuit with varying X_L Fig 4.3(b) : Locus of current vector I_L with variation of X_L

Series RC circuit with varying Reactance X_C:

Refer to the series RC circuit shown in the figure (a) below with constant R and varying X_C. The current I_C leads the applied voltage V by a phase angle $\theta = \tan^{-1}(X_C/R)$ for a given value of R as shown in the figure (b) below. When $X_C = 0$ we can see that the current is maximum equal to V/R and lies along the V axiswith phase angle equal to 0^0. When X_C is increased from zero to infinity the current gradually reduces from V/R to 0 and phase angle increases from 0^0 to -90^0. As can be seen from the figure, the tip of the current vector traces the path of a semicircle with its diameter along the +ve V axis but now on to its left side.



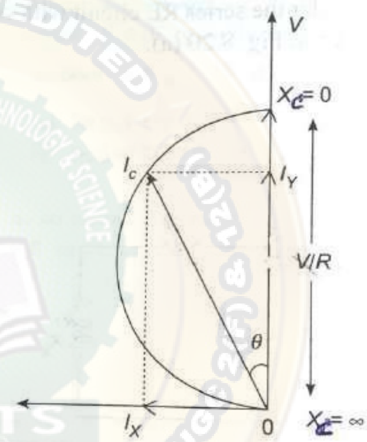
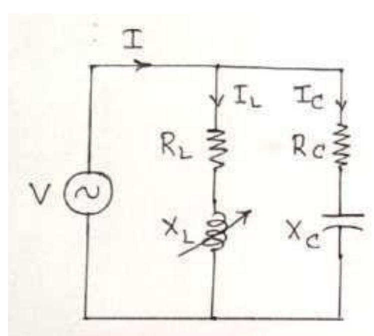


Fig4.4(a):SeriesRCcircuitwithvaryingX_C Fig4.4(b):LocusofcurrentvectorI_C with variation of X_C

Parallel LC circuits:

Parallel LC circuit along with its internal resistances as shown in the figures below is considered here for drawing the locus diagrams. As can be seen, there are two branch currents I_C and I_L along with the total current I. Locus diagrams of the current I_L or I_C (depending on which arm is varied)and the total current I are drawn by varying R_L, R_C, X_L and X_C one by one.

Varying X_L:



The current I_C through the capacitor is constant since R_C and C are fixed and it leads the voltage vector OV by an angle $\theta_C = \tan^{-1}(X_C/R_C)$ as shown in the figure (b). The current I_L through the inductance is the vector OI_L . It's amplitude is maximum and equal to V/R_L when X_L is zero and it is in phase with the applied voltage V. When X_L is increased from zero to infinity it's amplitude decreases to zero and phase will be lagging the voltage by 90^0. In between, the phase angle will be lagging the voltage V by an angle $\theta_L = \tan^{-1}(X_L/R_L)$. The locus of the current vector I_L is a semicircle with a diameter of length equal to V/R_L. Note that this is the same locus what we got earlier for the series RL circuit with X_L varying except that here V is shown horizontally.

Now, to get the locus of the total current vector OI we have to add vectorially the currents I_C and I_L . We know that to get the sum of two vectors geometrically we have to place one of the vectors staring point (we will take varying amplitude vector I_L) at the tip of the other vector (we will take constant amplitude vector I_C) and then join the start of fixed vector I_C to the end of varying vector I_L. Using this principle we can get the locus of the total current vector OI by shifting the I_L semicircle starting point O to the end of current vector OI_C keeping the two diameters parallel. The resulting semi circle I_CIB_T shown in the figure in dotted lines is the locus of the total current vector OI.

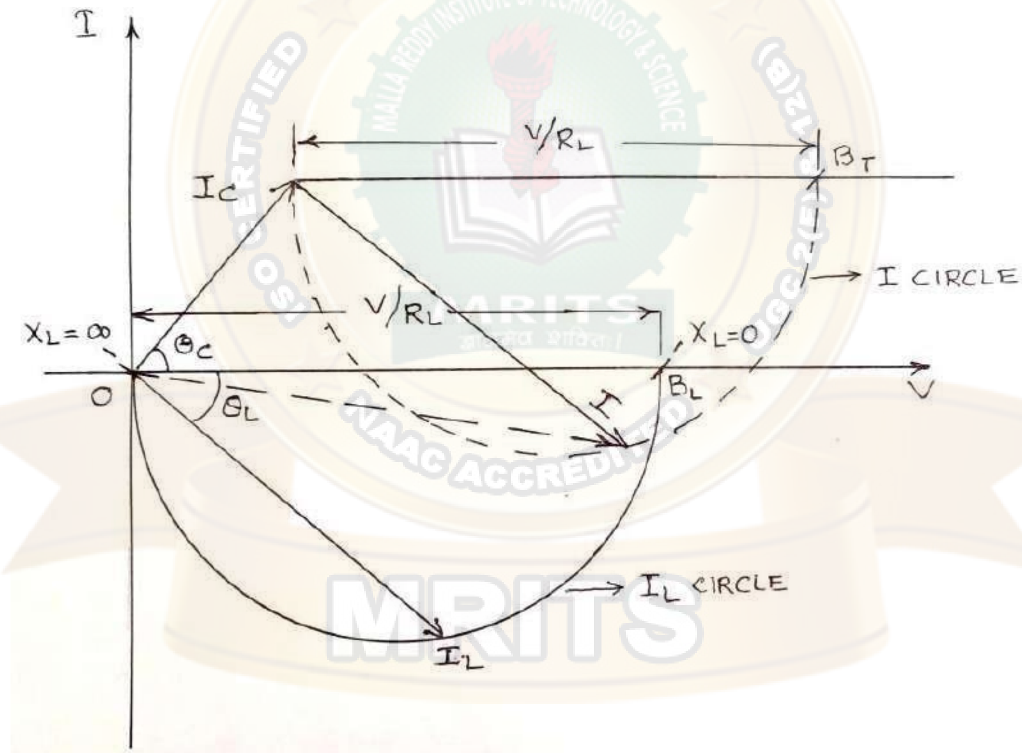


Fig 4.5(b): Locus of current vector I in Parallel LC circuit when X_L is varied from 0 to ∞

Varying X_C:

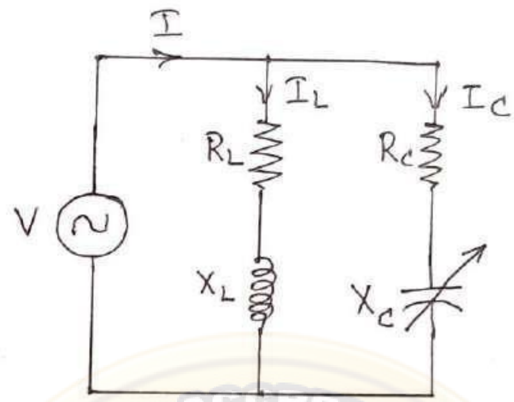


Fig.4.6(a) parallel LC circuit with Internal Resistances R_L and R_C in series with L (fixed) and C(Variable) respectively.

The current I_L through the inductor is constant since R_L and L are fixed and it lags the voltage vector OVby an angle $\theta_L = \tan^{-1}(X_L/R_L)$ as shown in the figure (b). The current I_C through the capacitance is the vector OI_C. It's amplitude is maximum and equal to V/R_C when X_C is zero and it is in phase with the applied voltage V. When X_C is increased from zero to infinity it's amplitude decreases to zero and phase will be leading the voltage by 90^0. In between, the phase angle will be leading the voltage V by an angle $\theta_C = \tan^{-1}(X_C/R_C)$. The locus of the current vector I_C is a semicircle with a diameter of length equal to V/R_C as shownin the figure below. Note that this is the same locus what we got earlier for the series RC circuit with X_C varying except that here V is shownhorizontally.

Now, to get the locus of the total current vector OI we have to add vectorially the currents I_C and I_L. We know that to get the sum of two vectors geometrically we have to place one of the vectors staring point (we will take varying amplitude vector I_C)at the tip of the other vector (we will take constant amplitude vector I_L) and then join the start of the fixed vector I_L to the end of varying vector I_C. Using this principle we can get the locus of the total current vector OI by shifting the I_C semicircle starting point O to the end of current vector OI_L keeping the two diameters parallel. The resulting semicircle I_LIB_T shown in the figure in dottedlines is thelocus of the total current vector OI.

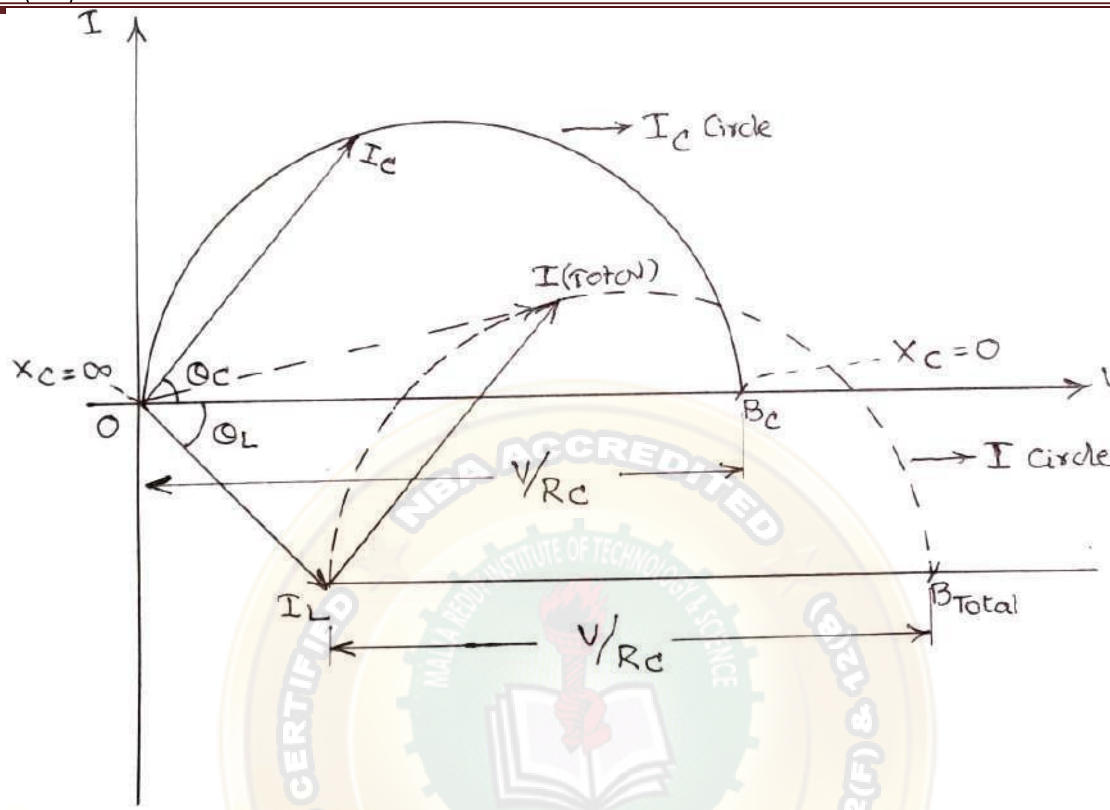


Fig4.6 (b) : Locus of current vector I in Parallel LC circuit when X_C is varied from 0 to ∞

Varying R_L:

The current I_C through the capacitor is constant since R_C and C are fixed and it leads the voltage vector OV by an angle $\theta_C = \tan^{-1}(X_C/R_C)$ as shown in the figure (b). The current I_L through the inductance is the vector OI_L . It's amplitude is maximum and equal to V/X_L when R_L is zero. Its phase will be lagging the voltage by 90^0. When R_L is increased from zero to infinity it's amplitude decreases to zero and it is in phase with the applied voltage V. In between, the phase angle will be lagging the voltage V by an angle $\theta_L = \tan^{-1}(X_L/R_L)$. The locus of the current vector I_L is a semicircle with a diameter of length equal to V/R_L. Note that this is the same locus what we got earlier for the series RL circuit with R varying except that here V is shown horizontally.

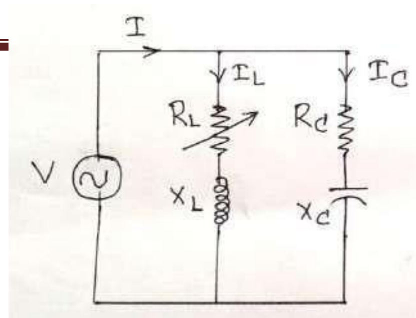


Fig. 4.7(a)parallelLC circuitwithInternalResistances R_L(Variable) andR_C(fixed) inseries with L and C respectively.

Now, to get the locus of the total current vector OI we have to add vectorially the currents I_C and I_L. We knowthat to get the sum of two vectors geometrically we have to place one of the vectors staring point (we will take varying amplitude vector I_L)at the tip of the other vector (we will take constant amplitude vector I_C)and then join the start of fixed vector I_C to the end of varying vector I_L. Using this principle we can get the locus of the total current vector OI by shifting the I_L semicircle starting point O to the end of current vector OI_C keeping the two diameters parallel. The resulting semicircle I_CIB_T shown in the figure in dotted lines is the locus of the total current vector OI.

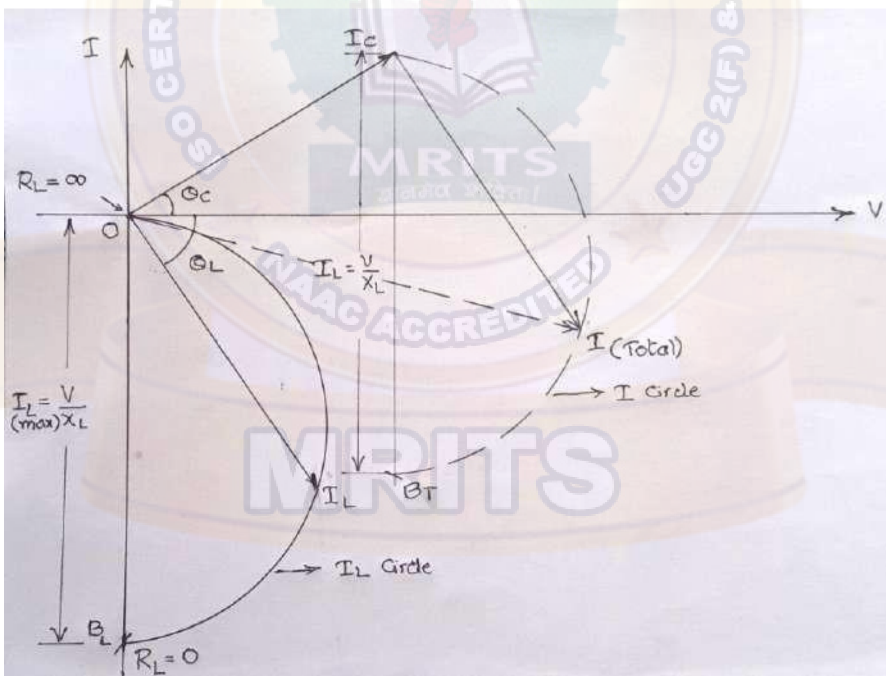


Fig 4.7(b) : Locus of current vector I in Parallel LC circuit when R_L is varied from 0 to ∞

Varying R_C:

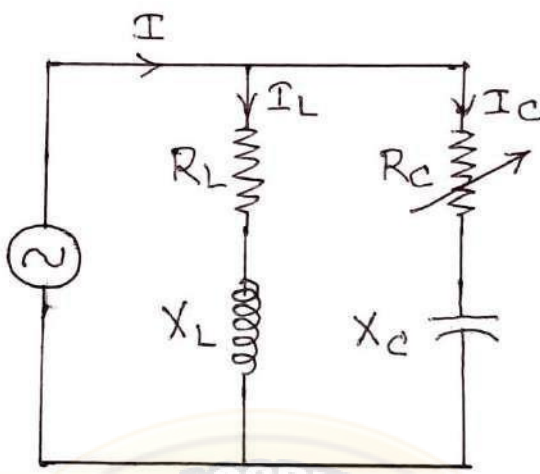


Fig. 4.8(a) parallel LC circuit with Internal Resistances R_L (fixed) and R_C (Variable) in series

with L and C respectively.

The current I_L through the inductor is constant since R_L and L are fixed and it lags the voltage vector OV by an angle $\theta_L = \tan^{-1}(X_L/R_L)$ as shown in the figure (b). The current I_C through the capacitance is the vector OI_C . It's amplitude is maximum and equal to V/X_C when R_C is zero and its phase will be leading thevoltage by 90^0 . When R_C is increased from zero to infinity it's amplitude decreases to zero and it will be in phase with the applied voltage V. In between, the phase angle will be leading the voltage V by an angle $\theta_C = \tan^{-1}(X_C/R_C)$. The locus of the current vector I_C is a semicircle with a diameter of length equal to V/X_C as shown in the figure below. Note that this is the same locus what we got earlier for the series RC circuit with R varying except that here V is shown horizontally.

Now, to get the locus of the total current vector OI we have to add vectorially the currents I_C and I_L . We know that to get the sum of two vectors geometrically we have to place one of the vectors staring point (we will take varying amplitude vector I_C)at the tip of the other vector (we will take constant amplitude vector I_L) and then join the start of the fixed vector I_L to the end of varying vector I_C. Using this principle we can get the locus of the total current vector OI by shifting the I_C semicircle starting point O to the end of current vector OI_L keeping the two diameters parallel. The resulting semicircle $I_L IB_T$ shown in the figure in dotted lines is the locus of the total current vector OI.

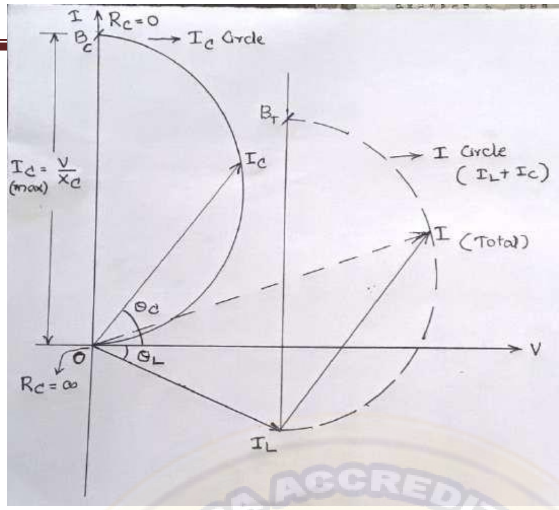


Fig 4.8(b) : Locus of current vector I in Parallel LC circuit when R_C is varied from 0 to ∞

Resonance:

Series RLC circuit:

The impedance of the series RLC circuit shown in the figure below and the current I through the circuit are given by :

$Z= R+j\omega L+1/j\omega C = R+j(\omega L-1/\omega C)$ $I = Vs/Z$

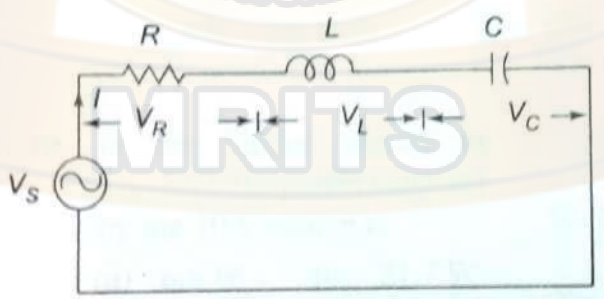


Fig 4.9: Series RLC circuit

The circuit is said to be in resonance when the Inductive reactance is equal to the Capacitive reactance. i.e. **$X_L = X_C$ or $\omega L = 1/\omega C$.** (i.e. Imaginary of the impedance is zero) The frequency at which the resonance occurs is called resonant frequency. In the resonant condition when $X_L = X_C$ they cancel with each other since they are in phase opposition(180^0 out of phase) and net impedance of the circuitis purely resistive. In this condition the magnitudes of voltages across

the Capacitance and the Inductance are also equal to each other but again since they are of opposite polarity they cancel with each other and the entire applied voltage appears across the Resistance alone.
Solving for the resonant frequency from the above condition of Resonance : **ωL = 1/ωC**

$$2\pi f_r L = 1/2\pi f_r C$$

$$f_r^2 = 1/4\pi^2 LC \quad \text{and} \quad f_r = 1/2\pi\sqrt{LC}$$

In a series RLC circuit, resonance may be produced by varying L or C at a fixed frequency or by varying frequency at fixed L and C.

Reactance, Impedance and Resistance of a Series RLC circuit as a function of frequency:

From the expressions for the Inductive and capacitive reactance we can see that when the frequencyis zero, capacitance acts as an open circuit and Inductance as a short circuit. Similarly when the frequency is infinity inductance acts as an open circuit and the capacitance acts as a short circuit. The variation of Inductive and capacitive reactance along with Resistance R and the Total Impedance are shown plotted in the figure below.
As can be seen, when the frequency increases from zero to ∞ Inductive reactance X_L (directly proportional to ω) increases from zero to ∞ and capacitive reactance X_C (inversely proportional to ω) decreases from −∞ to zero. Whereas, the Impedance decreases from ∞ to Pure Resistance R as the frequency increases from zero to f_r (as capacitive reactance reduces from −∞ and becomes equal to Inductive reactance) and then increases from R to ∞ as the frequency increases from f_r to ∞ (as inductive reactance increases from its value at resonant frequency to ∞)

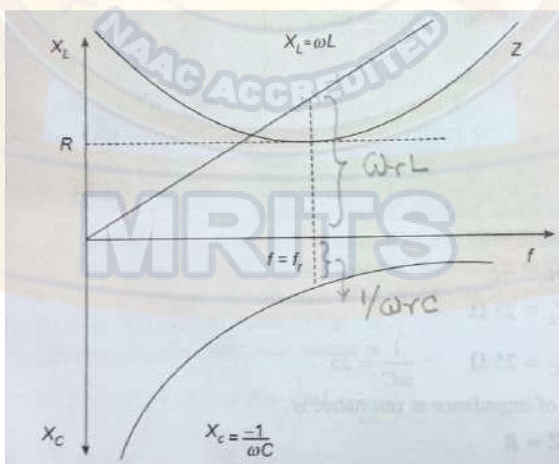


Fig 4.10: Reactance and Impedance plots of a Series RLC circuit

Phase angle of a Series RLC circuit as a function of frequency:

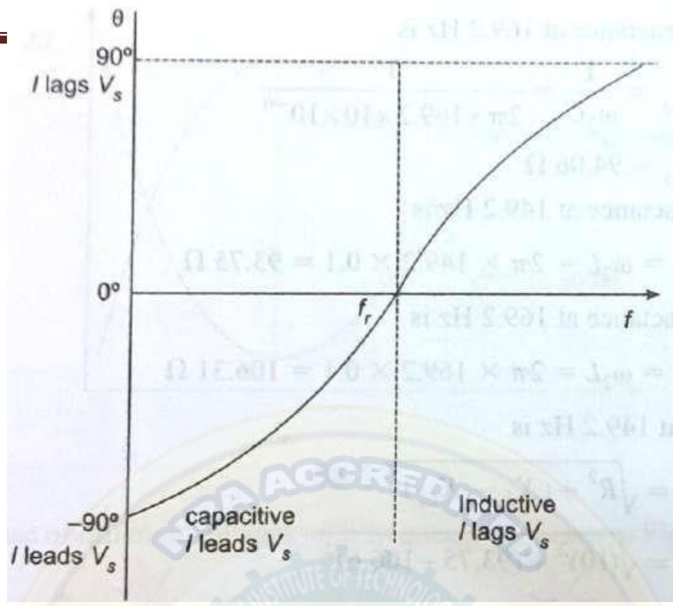


Fig4.11 : Phase plot of a Series RLC circuit

The following points can be seen from the Phase angle plot shown in the figure above:

- At frequencies below the resonant frequency capacitive reactance is higher than the inductivereactanceand hencethephaseangleofthecurrentleadsthevoltage.
- As frequency increases from zero to f_r the phase angle changes from -90^0 to zero.
- At frequencies above the resonant frequency inductive reactance is higher than the capacitivereactanceand hencethephaseangleofthecurrentlagsthevoltage.
- As frequency increases from f_r and approaches ∞, the phase angle increases from zero and approaches 90^0

Band width of a Series RLC circuit:

The band width of a circuit is defined as the Range of frequencies between which the output power is half of or 3 db less than the output power at the resonant frequency. These frequencies are called the cutoff frequencies, 3db points or half power points. But when we consider the output voltage or current, the range of frequencies between which the output voltage or current falls to 0.707 times of the value at the resonant frequency is called the Bandwidth BW. This is because voltage/current are related to power by a factor of √ 2 and when we are consider √ 2 times less it becomes 0.707. But still these frequencies are called as cutoff frequencies, 3db points or half power points. The lower end frequency is called ***lower cutoff frequency*** and the higher end frequency is called upper ***cutoff frequency.***

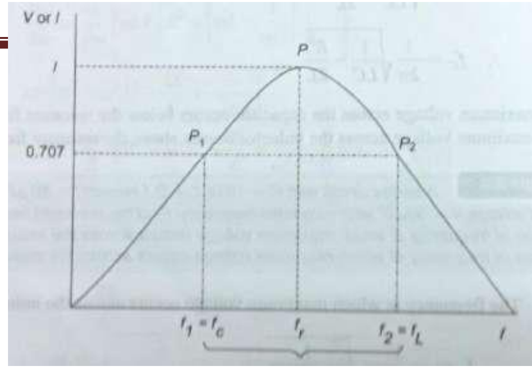


Fig 4.12: Plot showing the cutoff frequencies and Bandwidth of a series RLC circuit

Derivation of an expression for the BW of a series RLC circuit:

We know that BW = $f_2 - f_1$ Hz

If the current at points P_1 and P_2 are 0.707 ($1/\sqrt{2}$) times that of I_{max} (current at the resonant frequency) then the Impedance of the circuit at points P_1 and P_2 is $\sqrt{2}$ R (i.e. $\sqrt{2}$ times the impedance at f_r)

But Impedance at point P_1 is given by: $Z = \sqrt{R^2 + (1/\omega_1 C - \omega_1 L)^2}$ and equating this to $\sqrt{2}R$ we get : $(1/\omega_1 C) - \omega_1 L = R$ ------ (1)

Similarly Impedance at point P_2 is given by: $Z = \sqrt{R^2 + (\omega_2 L - 1/\omega_2 C)^2}$ and equating this to $\sqrt{2}R$ we get: $\omega_2 L - (1/\omega_2 C) = R$ ------ (2)

Equating the above equations (1) and (2) we get:

$$1/\omega_1 C - \omega_1 L = \omega_2 L - 1/\omega_2 C$$

Rearranging we get $L(\omega_1+\omega_2) = 1/C[(\omega_1+\omega_2)/\omega_1\omega_2]$ i.e $\omega_1\omega_2 = 1/LC$

But we already know that for a series RLC circuit the resonant frequency is given by $\omega_r^2 = 1/LC$ Therefore: $\omega_1\omega_2 = \omega_r^2$ ---- (3) and $1/C = \omega_r^2 L$ (4)

Next adding the above equations (1) and (2) we get:

$$1/\omega_1 C - \omega_1 L + \omega_2 L - 1/\omega_2 C = 2R$$

$$(\omega_2 - \omega_1)L + (1/\omega_1 C - 1/\omega_2 C) = 2R$$

$$(\omega_2 - \omega_1)L + 1/C[(\omega_2 - \omega_1)/\omega_1\omega_2) = 2R \quad \text{------} \quad (5)$$

Using the values of $\omega_1\omega_2$ and $1/C$ from equations (3) and (4) above into equation (5) above we get: $(\omega_2 - \omega_1)L + \omega_r^2 L[(\omega_2 - \omega_1)/\omega_r^2) = 2R$

 $2L(\omega_2 - \omega_1) = 2R$ i.e. $(\omega_2 - \omega_1) = R/L$ and $(f_2 - f_1) = R/2\pi L$ ----- (6)

OrfinallyBandwidth **$BW = R/2\pi L$** -- (7)

Since f_r lies in the centre of the lower and upper cutoff frequencies f_1 and f_2 using the above equation (6) we can get:

$$f_1 = f_r - R/4\pi L \qquad ------ \quad (8)$$

$$f_2 = f_r + R/4\pi L \qquad ------ \quad (9)$$

Further by dividing the equation (6) above by f_r on both sides we get another **important** relation: **$(f_2 - f_1) / f_r = R/2\pi f_r L$ or $BW / f_r = R/2\pi f_r L$**---------------- (10)

Here an important property of a coil i.e. **Q factor** or **figure of merit** is defined as the ratio of the reactance to the resistance of a coil.

$$Q = 2\pi f_r L / R \text{-------------------------------}(11)$$

Now using the relation (11) we can rewrite the relation (10) as

$$Q = f_r / BW \text{---------------------------------}(12)$$

Quality factor of a series RLC circuit:

The quality factor of a series RLC circuit is defined as:

Q = Reactive power in Inductor (or Capacitor) at resonance /Average power at Resonance

Reactive power in Inductor at resonance = I^2X_L

Reactive power in Capacitor at resonance = I^2X_C

Average power at Resonance = I^2R

Herethepowerisexpressedintheform I^2X (notasV^2/X)sinceIiscommonthroughR.LandC in the series RLC circuitanditgetscancelledduringthesimplification.

Therefore **$Q = I^2X_L / I^2R = I^2X_C / I^2R$**

i.e. $Q = X_L / R = \omega_r L / R$ -------------------------------------(1)

Or $Q = X_C / R = 1/\omega_r RC$---------------------------------- (2)

From these two relations we can also define Q factor as :

Q = Inductive (or Capacitive) reactance at resonance / Resistance

Substituting the value of $\omega_r = 1/\sqrt{LC}$ in the expressions (1) or (2) for **Q** above we can get the value of **Q** in terms of **R, L,C** as below.

$$Q = (1/\sqrt{LC})\, L / R \quad = \quad (1/R)\,(\sqrt{L/C})$$

Selectivity:

Selectivity of a series **RLC** circuit indicates how well the given circuit responds to a given resonant frequency and how well it rejects all other frequencies. i.e. the selectivity is directly proportional to **Q** factor.A circuit with a good selectivity (or a high **Q** factor) will have maximum gain at the resonant frequency and will have minimum gain at other frequencies .i.e. it will have very low band width. This is illustrated in the figure below.

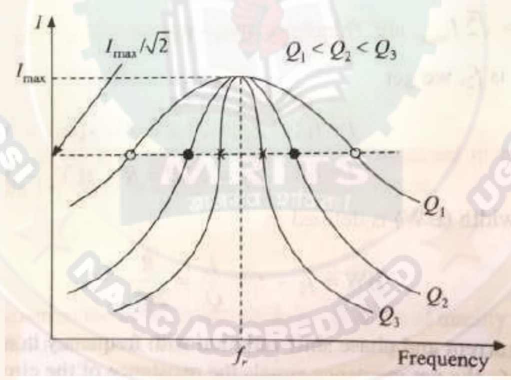


Fig 4.13: Effect of quality factor on bandwidth Voltage Magnification at resonance:

At resonance the voltages across the Inductance and capacitance are much larger than the applied voltage in a series RLC circuit and this is called voltage magnification at Resonance. The voltage magnification is equal to the **Q** factor of the circuit. This is proven below.

If we take the voltage applied to the circuit as **V** and the current through the circuit at resonanceas **I** then

The voltage across the inductance **L** is: $V_L = IX_L = (V/R)\, \omega_r L$ and

The voltage across the capacitance **C** is: $V_C = IX_C = V/R\, \omega_r C$

But we know that the **Q** of a series RLC circuit = $\omega_r L/ R = 1/R\, \omega_r C$ Using these relations in the expressions for V_L and V_C given above we get $V_L = VQ$ and $V_C = VQ$

The ratio of voltage across the Inductor or capacitor at resonance to the applied voltage in a series RLC circuit is called Voltage magnification and is given by

$$\text{Magnification} = Q = V_L/V \quad \text{or } V_C / V$$

Important points In Series RLC circuit at resonant frequency :

- The impedance of the circuit becomes purely resistive and minimum i.e **Z = R**
- The current in the circuit becomes maximum
- The magnitudes of the capacitive Reactance and Inductive Reactance become equal
- The voltage across the Capacitor becomes equal to the voltage across the Inductor at resonance and is **Q** times higher than the voltage across the resistor

Bandwidth and Q factor of a Parallel RLC circuit:

Parallel RLC circuit is shown in the figure below. For finding out the **BW** and **Q** factor of a parallel RLC circuit, since it is easier we will work with Admittance , Conductance and Susceptance instead of Impedance, Resistance and Reactance like in series RLC circuit.

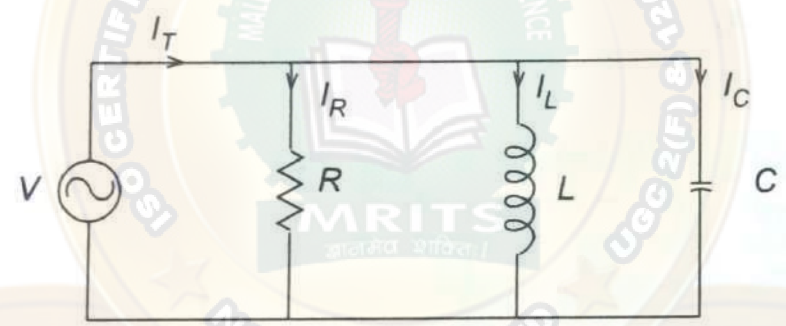


Fig 4.14 : Parallel RLC circuit

Then we have the relation: $Y = 1/Z = 1/R + 1/j\omega L + j\omega C = 1/R + j (\omega C - 1/\omega L)$

For the parallel RLC circuit also, at resonance, the imaginary part of the Admittance is zero and hence the frequency at which resonance occurs is given by: $\omega_r C - 1/\omega_r L = 0$. From this we get : $\omega_r C = 1/\omega_r L$ and $\omega_r = 1/\sqrt{LC}$
which is the same value for ω_r as what we got for the series RLC circuit.

At resonance when the imaginary part of the admittance is zero the ***admittance*** becomes ***minimum***.(i.e ***Impedance*** becomes ***maximum*** as against Impedance becoming minimum in series RLC circuit) i.e. Current becomes minimum in the parallel RLC circuit at resonance (as against current becoming maximum in series RLC circuit) and increases on either side of the resonant frequency as shown in the figure below.

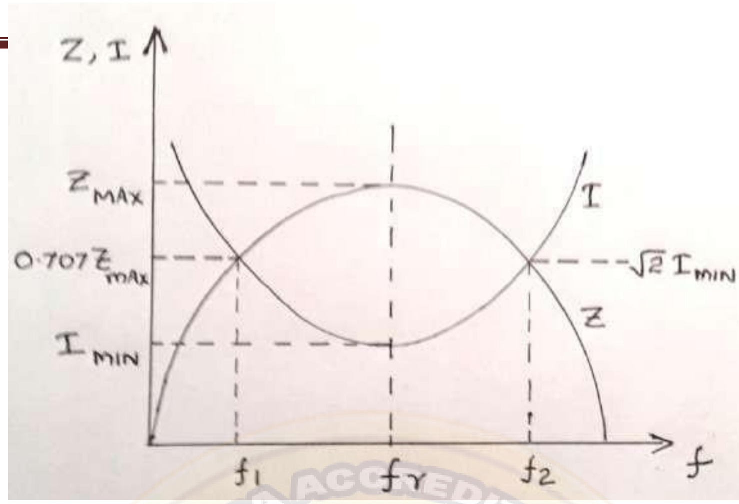


Fig 4.15: Variation of Impedance and Current with frequency in a Parallel RLC circuit

Here also the BW of the circuit is given by BW = f_2-f_1 where f_2 and f_1 are still called the upper and lower cut off frequencies but they are 3db higher cutoff frequencies since we notice that at these cutoff frequencies the amplitude of the current is √2 times higher than that of the amplitude of current at the resonant frequency.

The BW is computed here also on the same lines as we did for the series RLC circuit:

If the current at points P_1 and P_2 is √ 2 (3db) times higher than that of I_{min}(current at the resonant frequency) then the admittance of the circuit at points P_1 and P_2 is also √ 2 times higher than the admittance at f_r)

But amplitude of admittance at point P_1 is given by: $Y = \sqrt{1/R^2 + (1/\omega_1 L - \omega_1 C)^2}$ andequating this to $\sqrt{2}/R$ we get

$1/\omega_1 L - \omega_1 C = 1/R$ ---------------- (1)

Similarly amplitude of admittance at point P_2 is given by: $Y = \sqrt{1/R^2 + (\omega_2 C - 1/\omega_2 L)^2}$ and equating thisto $\sqrt{2}/R$ we get

$\omega_2 C - 1/\omega_2 L = 1/R$ ------------ (2)

Equating LHS of (1) and (2) and further simplifying we get

$$1/\omega_1 L - \omega_1 C = \omega_2 C - 1/\omega_2 L$$

$$1/\omega_1 L + 1/\omega_2 L = \omega_1 C + \omega_2 C$$

$$1/L\,[(\omega_1 + \omega_2)/\,\omega_1\omega_2] = (\omega_1 + \omega_2)C$$

$$1/LC = \omega_1\omega_2$$

Next adding the equations (1) and (2) above and further simplifying we get

$$1/\omega_1 L - \omega_1 C + \omega_2 C - 1/\omega_2 L = 2/R\ (\omega_2 C -$$

$$\omega_1 C) + (1/\omega_1 L - 1/\omega_2 L) = 2/R$$

$$(\omega_2 - \omega_1)C + 1/L[(\omega_2 - \omega_1)/\omega_1\omega_2] = 2/R$$ Substituting

the value of $\omega_1\omega_2 = 1/LC$

$$(\omega_2 - \omega_1)C + LC/L\ [(\omega_2 - \omega_1)] = 2/R(\omega_2$$

$$-\omega_1)C + C[(\omega_2 - \omega_1)] = 2/R\ 2\ C$$

$$[(\omega_2 - \omega_1)] = 2/R$$

$$\text{Or}[(\omega_2 - \omega_1)] = 1/RC$$

From which we get the band width **BW = $f_2 - f_1 = 1/2\pi RC$**

Dividing both sides by f_r we get: **$(f_2-f_1)/f_r = 1/2\pi f_r RC$ ---------- (1)**

Quality factor of a Parallel RLC circuit:

The quality factor of a Parallel RLC circuit is defined as:

Q = Reactive power in Inductor (or Capacitor) at resonance /Average power at Resonance

Reactive power in Inductor at resonance = V^2/X_L

Reactive power in Capacitor at resonance = V^2/X_C

Average power at Resonance = V^2/R

Here the power is expressed in the form V^2/X (not as I^2X as in series circuit) since V is common across R,L and C in the parallel RLC circuit and it gets cancelled during the simplification.

Therefore **$Q = (V^2/X_L) / (V^2/R) = (V^2/X_C) / (V^2/R)$**

i.e. **$Q = R/X_L = R/\omega_r L$** ----- (1)

Or **$Q = R/X_C = \omega_r RC$** ----- (2)

From these two relations we can also define **Q** factor as :

Q = Resistance /Inductive (or Capacitive) reactance at resonance

Substituting the value of $\omega_r = 1/\sqrt{LC}$ in the expressions (1) or (2) for **Q** above we can get the value of **Q** in terms of R, L,C as below.

$$Q = (1/\sqrt{LC})\,RC \quad = \quad R(\sqrt{C/L})$$

Further using the relation $Q = \omega_r RC$ (equation 2 above) in the earlier equation (1) we got in BW viz. $(f_2-f_1)/f_r = 1/2\pi f_r RC$ we get: $(f_2-f_1)/f_r = 1/Q$ or $Q = f_r/(f_2-f_1) = f_r/BW$

i.e. In Parallel RLC circuit also the Q factor is inversely proportional to the BW.

Admittance, Conductance and Susceptance curves for a Parallel RLC circuit as a function of frequency :

- The effect of varying the frequency on the Admittance, Conductance and Susceptance of a parallel circuit is shown in the figure below.
- Inductive susceptance B_L is given by $B_L = -1/\omega L$. It is inversely proportional to the frequency ω and is shown in the in the fourth quadrant since it is negative.
- Capacitive susceptance B_C is given by $B_C = \omega C$. It is directly proportional to the frequency ω and is shown in the in the first quadrant as OP .It is positive and linear.
- Net susceptance $B = B_C - B_L$ and is represented by the curve **JK**. As can be seen it is zero at the resonant frequency f_r
- The conductance $G = 1/R$ and is constant
- The total admittance **Y** and the total current **I** are minimum at the resonant frequency as shown by the curve **VW**

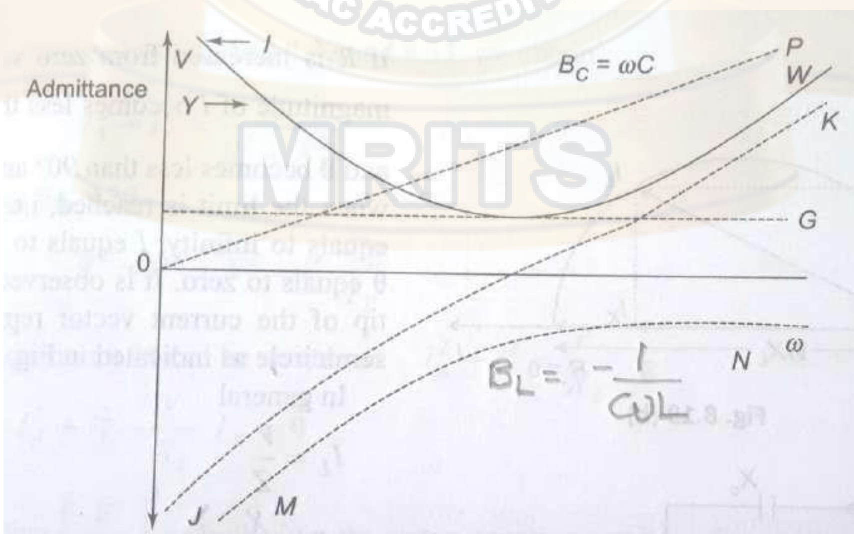


Fig4.16:Conductance,SusceptanceandAdmittanceplotsofaParallelRLCcircuit Current magnification in a Parallel RLC circuit:

Justas voltage magnification takesplace across the capacitance and Inductanceattheresonant frequency in a series RLC circuit, current magnification takes place in the currents through the capacitance and Inductance at the resonant frequency in a Parallel RLC circuit. This is shown below.

Voltage across the Resistance $= V = IR$

CurrentthroughtheInductanceatresonance$I_L = V/\omega_r L = IR/\omega_r L = I \cdot R/\omega_r L = IQ$ **Similarly**

CurrentthroughtheCapacitanceatresonance$I_C = V/(1/\omega_r C) = IR/(1/\omega_r C) = I(R\omega_r C) = IQ$

From which we notice that the quality factor $Q = I_L / I$ or I_C / I and that the current through the inductance andthecapacitanceincreasesby **Q** timesthatofthecurrentthroughtheresistorat resonance. .

Important points In Parallel RLC circuit at resonant frequency :

- Theimpedanceofthecircuitbecomesresistiveandmaximum i.e $Z = R$
- Thecurrentinthecircuit becomesminimum
- ThemagnitudesofthecapacitiveReactanceandInductiveReactancebecomeequal
- Thecurrentthrough theCapacitorbecomesequalandoppositetothecurrentthroughthe Inductor atresonance andisQtimeshigherthanthecurrentthroughtheresistor

Magnetic Circuits:

Introduction to the Magnetic Field:

Magnetic fields are the fundamental medium through which energy is converted from one form to another in motors, generators and transformers. Four basic principles describe how magnetic fields are used in these devices.

0. Acurrent-carryingconductorproducesamagneticfieldinthearea aroundit.

Explained in Detail by Fleming's Right hand rule and Amperes Law.

1. Atimevaryingmagneticflux induces avoltageina coilof wireifitpassesthroughthatcoil. (basis of Transformeraction)

Explained in detail by the Faradays laws of Electromagnetic Induction.

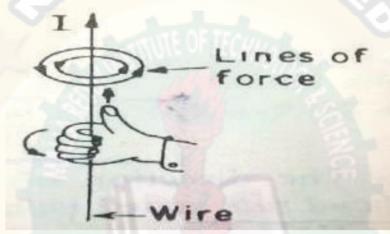
2. Acurrentcarryingconductorinthepresenceofamagneticfieldhasaforceinducedinit(Basis of Motoraction)
3. A moving wire in the presence of a magnetic field has a voltage induced in it (Basis of Generator action)

Wewillbestudyinginthisunitthefirsttwoprinciplesindetailandtheothertwoprinciplesin the next unit on DC machines.

Twobasiclawsgoverningtheproductionofamagneticfieldbyacurrentcarryingconductor:

The direction of the magnetic field produced by a current carrying conductor is given by the ***Flemings Right hand rule*** and its' amplitude is given by the ***Ampere's Law***.

Flemings right hand rule: *Holdtheconductor carrying thecurrentinyourright hand suchthat the Thumb points along the wire in the direction of the flow of current, then the fingers will encircle the wire along the lines of the Magnetic force.*



Ampere's Law : *The line integral of the magnetic field intensity H around a closed magnetic path is equal to the total current enclosed by the path.*

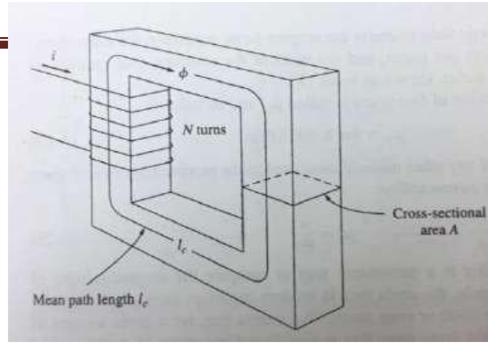
This is the basic law which gives the relationship between the Magnetic field Intensity H and the current I and is mathematically expressed as

$$\boldsymbol{H}.\boldsymbol{d}l = I_{net}$$

where **H** is the magnetic field intensity produced by the current I_{net} and ***dl*** is a differential element of lengthalongthepathofintegration.**H** ismeasuredin *Ampere-turns per meter*.

Important parameters and their relation in magnetic circuits :

- Consider a current carrying conductor wrapped around a ferromagnetic core as shown in the figure below.



- Applying Ampere's law, the total amount of magnetic field induced will be proportional to the amount of current flowing through the conductor wound with N turns around the ferromagnetic material as shown. Since the core is made of ferromagnetic material, it is assumed that a majority of the magnetic field will be confined to thecore.
- The path of integration in this case as per the Ampere's law is the mean path length of the core, l_c. The current passing withinthepathofintegrationI_{net} isthen **Ni**,sincethecoilof wirecuts thepathofintegration**N** timeswhilecarrying thecurrent**i**.HenceAmpere'sLawbecomes: $Hl_c = Ni$

Therefore $$H = Ni/l_c$$

- In this sense, **H** (Ampere turns per meter) is known as the effort required to induce a magnetic field. The strength ofthemagnetic field flux producedin the core also depends on the material of the core. Thus: $B = \mu H$ where

B = magnetic flux density [webers per square meter, or Tesla (T)]

μ= magnetic permeability of material (Henrys per meter)

H = magnetic field intensity (ampere-turns per meter)

- The constant **μ** may be further expanded to include ***relative permeability*** which can be defined as below:

$$\mu_r = \mu / \mu_o$$

where μ_o = permeability of free space (equal to that of air)

- Hence the permeability value is a combination of the relative permeability and the permeability of free space. The value of relative permeability is dependent upon the type of material used. The higher the amount permeability, thehigherthe amount of flux induced in the core. Relative permeability is a convenient way to compare the magnetizability of materials.
- Also, because the permeability of iron is so much higher than that of air, the majority of the flux in an iron core remains inside the core instead of travelling through the surrounding air, which has lower permeability. The small leakage fluxthatdoesleavetheironcoreisimportantin

determining the flux linkages between coils and the self-inductances of coils in transformers and motors.

- In a core such as shown in the figure above

$$B = \mu H = \mu Ni/l_c$$

Now, to measure the total flux flowing in the ferromagnetic core, consideration has to be made in terms of its cross sectional area (CSA). Therefore:

$\Phi = B.dA$ where: **A** = cross sectional area throughout the core.

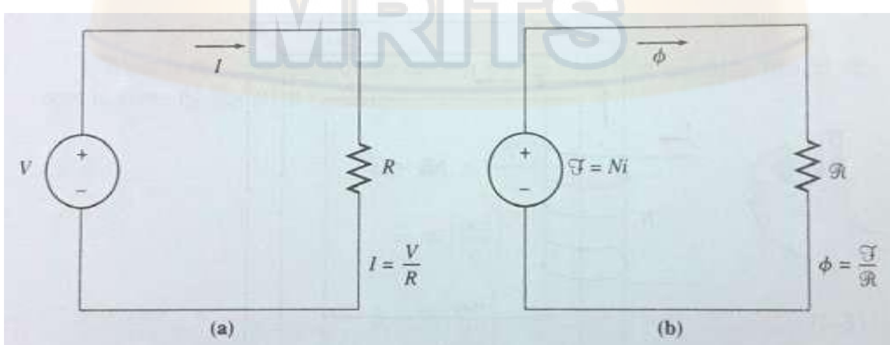
Assuming that the flux density in the ferromagnetic core is constant throughout hence the equation simplifies to: $\Phi = B.A$

Taking the previous expression for **B** we get $\Phi = \mu NiA/l_c$

Electrical analogy of magnetic circuits:

The flow of magnetic flux induced in the ferromagnetic core is analogous to the flow of electric current in an electrical circuit hence the name magnetic circuit.

The analogy is as follows:



(a) ElectricCircuit **(b) Electrical Analogy of Magnetic Circuit**

Referring to the magnetic circuit analogy, **F** is denoted as **magnetomotive force** (mmf) which is similar to Electromotive force in an electrical circuit (emf). Therefore, we can say that **F** is the force which pushes magnetic flux around a ferromagnetic core with a value of **Ni** (refer to ampere's law). Hence **F** is measured in ampere turns. Hence the magnetic circuit equivalent equation is asshown:

$$F = \emptyset.R \text{ (similar to } V=IR)$$

We already have the relation **Φ = μ NiA/l** and using this we get **R = F / Φ = Ni/ Φ**

$$R = Ni /(\mu NiA/l) = l/ \mu A$$

- The polarity of the mmf will determine the direction of flux. To easily determine the direction of flux, the ***'right hand curl'*** rule is applied:

Whenthedirectionofthecurledfingersindicatesthedirectionofcurrentflowthe resulting thumb direction willshowthemagneticfluxflow.

- The element of **R** in the magnetic circuit analogy is similar in concept to the electrical resistance. It is basically the measure of material resistance to the flow of magnetic flux. **Reluctance** in this analogy obeys the rule of electrical resistance (Series and Parallel Rules). Reluctance is measured in ***Ampere-turns per weber***.
- The inverse of electrical resistance is conductance which is a measure of conductivity of a material. Similarly theinverse of reluctance is known as **permeance P** which represents the degree to which the material permits theflowofmagneticflux.

- By using the magnetic circuit approach, calculations related to the magnetic field in a ferromagnetic materialare simplifiedbutwithalittleinaccuracy.

Equivalent Reluctanceof a series Magneticcircuit : $R_{eqseries} = R_1 + R_2 + R_3 + ...$

Equivalent Reluctance of a Parallel Magnetic circuit:

$1/R_{eqparallel} = 1/R_1 + 1/R_2 + 1/R_3 + ...$Electromagnetic Induction and Faraday's law – Induced Voltage from a Time-Changing Magnetic Field:

Faraday's Law:

Whenever a varying magnetic flux passes through a turn of a coil of wire, voltage will be induced in theturnofthewirethatisdirectlyproportionaltotherateofchangeof theflux linkage with the turn of the coil of wire.

$$e_{ind} \; \alpha \; -d\text{Ø}/dt$$

$$e_{ind} = -k.\, d\text{Ø}/dt$$

The negative sign in the equation above is in accordance to **Lenz' Law** which states:

The direction of the induced voltage in the turn of the coil is such that if the coil is short circuited, itwould produce acurrentthat wouldcause aflux which opposestheoriginal change of flux.

And **k** is the constant of proportionality whose value depends on the system of units chosen. In the SI systemofunits **k=1** andtheaboveequationbecomes:

$$e_{ind} = -\, d\text{Ø}/dt$$

Normallyacoilisusedwithseveralturnsandifthereare Nnumberofturnsinthecoilwiththe same amount of flux flowing through it then: $e_{ind} = -N\, d\text{Ø}/dt$

Change in the flux linkage NØ of a coil can be obtained in two ways:

1. Coil remains stationary and flux changes with time (Due to AC current like in Transformers and this is calledStatically induced e.m.f)
2. Magnetic flux remains constant and stationary in space, but the coil moves relative to the magnetic field so as to create a change in the flux linkage of the coil (Like in Rotating machines and this is a called Dynamically inducede.m.f.

Self inductance:

From the Faradays laws of Electromagnetic Induction we have seen that an e.m.f will be induced in a conductor when a time varying flux is linked with a conductor and the amplitude of the induced e.m.f is proportionaltotherateofchangeofthevaryingflux.

If the time varying flux is produced by a coil of **N** turns then the coil itself links with the time varying flux producedbyitselfandanemfwillbeinducedinthesamecoil.Thisiscalledself inductance .

The flux **Ø** produced by a coil of N turns links with its own N turns of the coil and hence the total flux linkage is equal to $N\text{Ø} = (\mu N^2 A / l)\, I$ *[using the expression $\Phi = \mu NiA/l$ we already develop*ed] Thus we see that the total magnetic flux produced by a coil of N turns and linked with itself is proportional to the current flowingthroughthecoili.e.

$$N\text{Ø} \propto I \text{ or } N\text{Ø} = L\, I$$

From the Faradays law of electromagnetic Induction, the self induced e.m.f for this coil of N turns is givenby:

$$e_{ind} = -N\, d\text{Ø}/dt = -L\, dI/dt$$

The constant of proportionality **L** is called the self Inductance of the coil or simply Inductance and its value is given by **L = (μ N² A / l)**. If the radius of the coil is **r** then:

$$L = (\mu N^2 \pi r^2 / l)\ i$$

From the above two equations we can see that Self Inductance of a coil can be defined as the flux produced per unit current i.e *Weber/Ampere* (equation1) or the induced emf per unit rate of change of current i.e *Volt-sec/Ampere* (equation 2)

The unit of Inductance is named after Joseph Henry as ***Henry*** and is given to these two combinations as :

$$1H = 1WbA^{-1} = 1VsA^{-1}$$

Self Inductance of a coil is defined as one Henry if an induced emf of one volt is generated when the current in the coil changes at the rate of one Ampere per second.

Henry is relatively a very big unit of Inductance and we normally use Inductors of the size of mH (10^{-3} H) or μH (10^{-3}H)

Mutual inductance and Coefficient of coupling:

In the case of Self Inductance an emf is induced in the same coil which produces the varying magnetic field. The same phenomenon of Induction will be extended to a separate second coil if it is located in the vicinity of the varying magnetic field produced by the first coil. Faradays law of electromagnetic Induction is equally applicable to the second coil also. A current flowing in one coil establishes a magnetic flux about that coil and also about a second coil nearby but of course with a lesser intensity. The time-varying flux produced by the first coil and surrounding the second coil produces a voltage across the terminals of the second coil. This voltage is proportional to the time rate of change of the current flowing through the first coil.

Figure (*a*) shows a simple model of two coils L_1 and L_2, sufficiently close together that the flux produced by a current $i_1(t)$ flowing through L_1 establishes an open-circuit voltage $v_2(t)$ across the terminals of L_2. ***Mutual inductance, M21***, is defined such that

$$v_2(t) = M_{21} di_1(t)/dt \text{ ------------------ [1]}$$

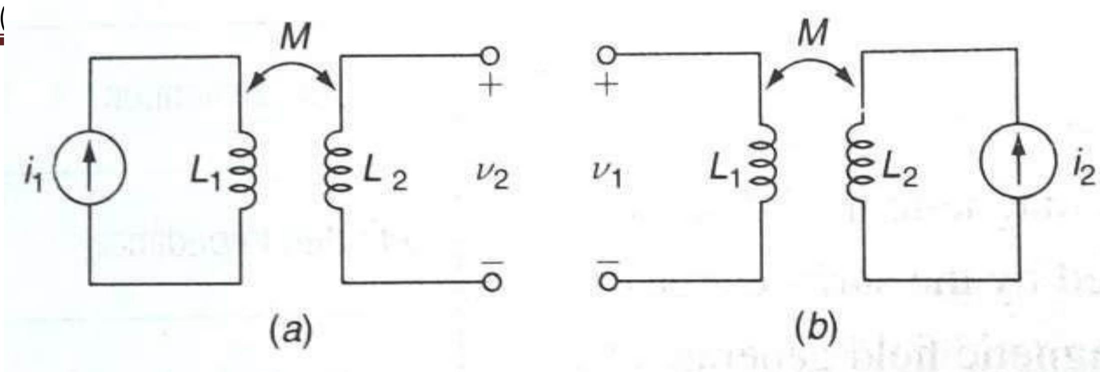


Figure4.17 (*a*)Acurrent *i*1through*L*1producesanopen-circuit voltage*v*2across*L*2. (*b*)A current*i*2 through*L*2producesanopen-circuitvoltage*v*1across*L*1.

The order of the subscripts on M_{21} indicates that a voltage response is produced at L_2 by a currentsource at L_1. If the system is reversed, as indicated

in fig.(*b*) then we have

$$v_1(t) = M_{12} di_2(t)/dt \text{ ------------------ [2]}$$

It can be proved that the two mutual inductances M_{12} and M_{21} are equal and thus, $M_{12} = M_{21} = M$. The existence of mutual coupling between two coils is indicated by a double-headed arrow, as shown inFig. (*a*)and (*b*)

Mutual inductance is measured in Henrys and, like resistance, inductance, and capacitance, is a positive quantity. The voltage $M\ di/dt$, however, may appear as either a positive or a negative quantity dependingon whether thecurrentisincreasingordecreasingataparticularinstantof time.

Coefficient of coupling k **:** Is given by the relation $\mathbf{M = k\sqrt{L_1 L_2}}$ and its value lies between 0 and

1. It can assume the maximum value of 1 when the two coils are wound on the same core such that flux produced by one coil completely links with the other coil. This is possible in well designed cores with high permeability. Transformers are designed to achieve a coefficient of coupling of 1.

Dot Convention:

The polarity of the voltage induced in a coil depends on the sense of winding of the coil. In the case of Mutualinductanceitisindicated byuseofamethodcalled"***dot convention".*** Thedot convention makes use of a large dot placed at one end of each of the two coils which are mutually coupled. Sign of the mutual voltage is determined as follows:

A current entering the dotted terminal of one coil produces an open circuit voltage with a positive voltage reference at the dotted terminal of the second coil.

Thus in Fig(*a*) i_1 enters the dotted terminal of L_1, v_2 is sensed positively at the dotted terminal of L_2, and $v_2 = M\, di_1/dt$.

It may not be always possible to select voltages or currents throughout a circuit so that the passive sign convention is everywhere satisfied; the same situation arises with mutual coupling. For example, it may be more convenient to represent v_2 by a positive voltage reference at the undotted terminal, as shown in Fig (*b*). Then $v_2 = -M\, di_1/dt$. Currents also may not always enter the dotted terminal as indicated by Fig (*c*) and (*d*). Then we note that:

A current entering the undotted terminal of one coil provides a voltage that is positively sensed at the undotted terminal of the second coil.

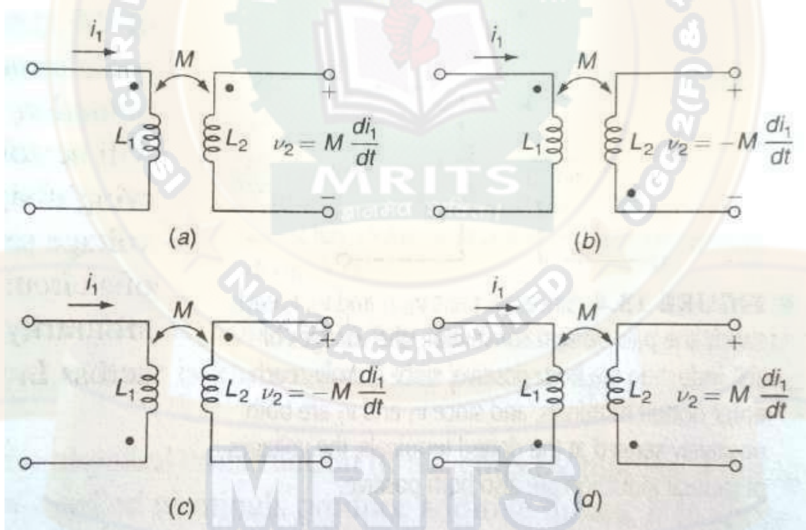


Figure 4.18 : (a) and (b) Current entering the dotted terminal of one coil produces a voltage that is sensed positively at the dotted terminal of the second coil. **(c) and (d)** Current entering the undotted terminal of one coil produces a voltage that is sensed positively at the undotted terminal of the second coil.

Important Concepts and formulae:

Resonance and Series RLC circuit:

$\omega_r^2 = \omega_1\omega_2 = 1/LC \therefore \omega_r = \sqrt{\omega_1\omega_2} = 1/\sqrt{LC} BW = R/2\pi L$

$Q = \omega_r L / R = 1/\omega_r RC$ andinterms of R,Land C = $(1/R)(\sqrt{L/C})$

$Q = f_r / BW$ i.e. inverselyproportionaltotheBW

Voltage magnification Magnification = $Q = V_L / V$ or V_C / V

Important points In Series RLC circuit at resonant frequency:

- Theimpedance ofthecircuit becomes purely resistive and minimum i.e **Z = R**
- The current in the circuit becomes maximum
- The magnitudes of the capacitive Reactance and Inductive Reactance become equal
- The voltage across the Capacitor becomes equal to the voltage across the Inductor at resonance and is **Q** times higher than the voltage across the resistor

Resonance and Parallel RLC circuit:

$\omega_r^2 = \omega_1\omega_2 = 1/LC \quad \therefore \omega_r = \sqrt{\omega_1\omega_2} = 1/\sqrt{LC}$ same as in series RLCcircuit

$BW = 1/2\pi RC$

$Q = R / \omega_r L = \omega_r RC$ and in terms of R, L and C = $R(\sqrt{C/L})$ [Inverseofwhatwegot in Series RLCcircuit]

$Q = f_r / BW$ In Parallel RLC also inversely proportional to the BW

Current Magnification = $Q = I_L / I$ or I_C / I

Important points In Parallel RLC circuit at resonant frequency :

- The impedance of the circuit becomes resistive and maximum i.e **Z = R**
- The current in the circuit becomes minimum
- The magnitudes of the capacitive Reactance and Inductive Reactance become equal
- Thecurrentthrough theCapacitorbecomesequalandoppositetothecurrentthroughthe Inductoratresonance andisQtimeshigherthanthe currentthroughtheresistor

Magnetic circuits :

Ampere's Law: $\oint \boldsymbol{H} . d\boldsymbol{l} = I_{net}$ and in the case of a simple closed magnetic pathofa ferromagneticmaterialitsimplifiesto ***Hl=Ni*** *or H = Ni/l*

Magneticfluxdensity:

$$B = \mu H$$

Magneticfieldintensity: $H = Ni/l$

Totalmagneticfluxintensity**:** $\text{Ø} = BA = \mu HA = \mu Ni\ A / l$

Reluctanceofthemagneticcircuit: **R = mmf/Flux = Ni/ Ø = l/μA**

Faradays law of electromagnetic Induction:

Selfinducede.m.fofacoilofNturnsisgivenby: $e_{ind} = -N dØ/dt = -L\, dI/dt$ whereListhe inductanceofthe coilofNturnswithradiusr andgivenby $L = (\mu N^2 \pi r^2 / l)\, i$

EquivalentReluctanceofaseriesMagneticcircuit: $R_{eqseries} = R_1 + R_2 + R_3 + \ldots$

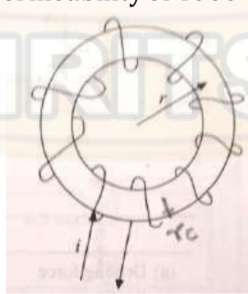
EquivalentReluctanceofaParallelMagneticcircuit: $1/R_{eqparallel} = 1/R_1 + 1/R_2 + 1/R_3 + \ldots$

Coefficientofcoupling*k* Isgiven bythe relation: $M = k\sqrt{L_1 L_2}$

Illustrative examples:

Example 1: A toroidal core of radius 6 cms is having 1000 turns on it. The radius of cross section of the core 1cm.Find the current required to establish a total magnetic flux of 0.4mWb.When

(a) The core isnonmagnetic
(b) The core is made of iron having a relative permeability of 4000



Solution:

This problem can be solved by the direct application of the following formulae we know in magnetic circuits:

B = Φ/A = μH and H = Ni/l

Where

B = magnetic flux density (Wb/mtr^2) Φ = Total magnetic flux (Wb))

A = Cross sectional area of the core(mtr^2) $\mu = \mu_r\mu_0$ = Permeability (Henrys/mtr) μ_r = Relative permeability of the material (Dimensionless)

μ_0 = free space permeability = $4\pi \times 10^{-7}$ Henrys/mtr

H = Magenetic field intensity AT/mtr N = Number of turns of the coil

i = Current in the coil (Amps) l = Length of the coil (mtrs)

from the above relations we can get **i** as

$i = Hl/N = (1/\mu)(\Phi/A)\, l / N = (1/\mu)(\Phi/N)\, l / A = (1/\mu)(\Phi/N) * 2\pi r_T / \pi r_C^2 = [2r_T \Phi / \mu N r_C^2]$

Where r_T is the radius of the toroid and r_C is the radius of cross section of the coil

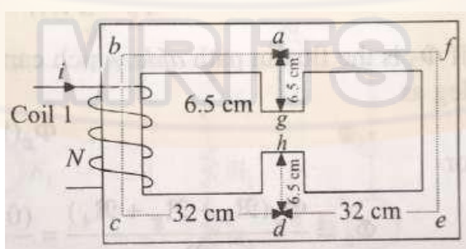
Now we can calculate the currents in the two cases by substituting the respective values. (a) Here $\mu = \mu_0$.

Therefore $i = (2 x 6 x 10^{-2} x 4 x 10^{-4}) / (4\pi x 10^{-7} x 1000 x 10^{-4}) = 380$ Amps

(b) Here $\mu = \mu_r\mu_0$. Therefore $i = (2 \times 6 \times 10^{-2} \times 4 \times 10^{-4}) / (4000 \times 4\pi \times 10^{-7} \times 1000 \times 10^{-4}) = 0.095$ Amps

Ex.2: (a) Draw the electrical equivalent circuit of the magnetic circuit shown in the figure below. The area of the core is 2x2 cm^2 .The length of the air gap is 1cm and lengths of the other limbs are shown in the figure. The relative permeability of the core is 4000.

(b) Find the value of the current **'i'** in the above example which produces a flux density of 1.2 Tesla in the airgap. The number f turns of the coil are 5000.



Solution: (a)

To draw the equivalent circuit we have to find the Reluctances of the various flux paths independently.

The reluctance of the path *abcd* is given by: R_1 = length of the path *abcd* / $\mu_r\mu_0 A$

$= (32x10^{-2})/(4\pi x10^{-7} x4000x4x10^{-4})$ $= 1.59 \times 10^5$ AT/Wb

The reluctance of the path *afed* is equal to the reluctance of the path *abcd* since it has the same length, same permeability and same cross sectional area. Thus $R_1 = R_2$

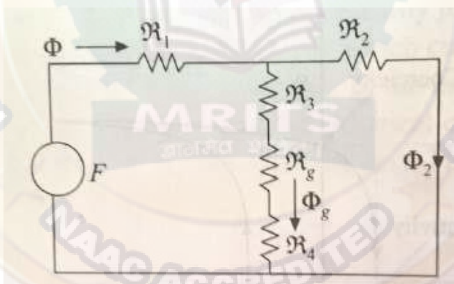
Similarly the reluctance of the path *ag* (R_3) is equal to that of the path *hd* (R_4) and can be calculated as:

$R_3 = R_4 = (6.5 \times 10^{-2}) / (4\pi \times 10^{-7} \times 4000 \times 4 \times 10^{-4}) = 0.32 \times 10^5$ AT/Wb

The reluctance of the air gap path *gh* R_G can be calculated as : R_G = length of the air gap path *gh*/$\mu_0 A$

(Here it is to be noted that μ is to be taken as μ_0 only and μ_r should not be included) $R_G = (1 \times 10^{-2}) / (4\pi \times 10^{-7} \times 4 \times 10^{-4}) = 198.94 \times 10^5$ AT/Wb

The equivalent electrical circuit is shown in the figure below with the values of the reluctances as given below the circuit diagram.



$R_1 = R_2 = 1.59 \times 10^5$ AT/Wb $\quad R_3 = R_4 = 0.32 \times 10^5$ AT/Wb $\quad R_G = 198.94 \times 10^5$ AT/Wb

Solution: (b) **This problem is solved in the following steps:**

1. **First the flux through the air gap Φ_G is found out.** The flux in the air gap Φ_G is given by the product of the Flux density in the air gap **B** and the cross sectional area of the core in that region **A** . Hence $\Phi_G = B.A = 1.2 \times 4 \times 10^{-4} = \mathbf{0.00048}$ **Wb**

It is to be noted here that the same flux would be passing through the reluctances R_3, R_G & R_4

2. **Next, the Flux in the path *afed* Φ_2 is to be found out .** This can be found out by noticing that the mmf across the reluctance R_2 is same as the mmf across the sum of the reluctances **R_3, R_G, and R_4** coming in parallel with **R_4**. Hence by equating them we get

$$\Phi_G (R_3 + R_G + R_4) = \Phi_2 R_2 \quad \text{fromwhichweget} \quad \Phi_2 = \Phi_G (R_3 + R_G + R_4) / R_2$$

Hence Φ_2 = $[0.00048 \times (0.32 + 198.94 + 0.32) \times 10^5] / 1.59 \times 10^5$ = **0.06025 Wb**

1. **Next , the total flux Φ flowing through the reluctance of the path abcd R_1 produced by the winding is to be found out.** This is the sum of the air gap flux Φ_G and the flux in the outer limb of the core Φ_2 :i.e $\Phi = \Phi_G + \Phi_2 = (0.00048 + 0.06025) =$ **0.0607 Wb**
2. **Next , The total mmf F given by F = Ni is to be found out .** This is also equal to the sum of the mmfs across the reluctances R_1 and R_2 [or $(R_3 + R_G + R_4)$] $= \Phi R_1 + \Phi_2 R_2$ from whichwe can get **'i'** as : **'i'** = $(\Phi R_1 + \Phi_2 R_2) / N = [0.0607 \times 1.59 \times 10^5 + 0.06025 \times 1.59 \times 10^5]/5000$ = 3.847 Amps

is = 3.847 Amps

UNIT-IV TRANSMISSION LINES-I

- Types of transmission lines
- Transmission line Parameters- Primary & Secondary Constants
- Transmission Line Equations
- Expressions for Characteristics Impedance
- Propagation Constant
- Phase and Group Velocities
- Infinite Line Concepts
- Lossless transmission line
- Distortion
- Condition for Distortionlessness transmission
- Minimum Attenuation
- Illustrative Problems.

TRANSMISSION LINE THEORY

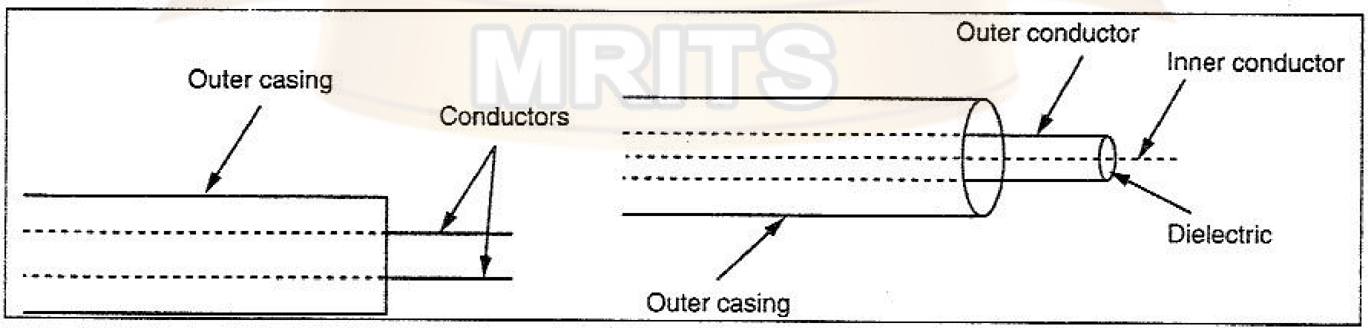
1.1. INTRODUCTION

The transfer of energy from one point to another takes place through either wave guides or transmission lines. Transmission lines always consist of atleast two separate conductors between which a voltage can exist, but the wave guides involve only one conductor; for example, a hollow rectangular or circular waveguide within which the wave propagates. Transmission lines are a means of conveying power from one point to another. There are two types of commonly used transmission lines.

1. Parallel wire (balanced) line
2. Coaxial (unbalanced) line

Parallel wire line : It is a common form of transmission line known as open wire line as shown in Fig. 1.1(a). It is employed where balanced properties are required. Telephone lines, line connecting between folded dipole antenna and TV receiver are good examples of parallel or balanced or open wire line. The parallel wire lines are not used for microwave transmission.

Coaxial line : Coaxial lines consist of inner and outer conductor spacers of dielectric as shown in Fig. 1.1(b). It is used when unbalanced properties are needed, as in the interconnection of a broadcast transmitter to its grounded antenna. It is employed at UHF and microwave frequencies.



(a) Parallel wire (balanced) line *(b) Coaxial (unbalanced) line*

Fig. 1.1. Transmission lines

1.2. TRANSMISSION LINE AS CASCADED T SECTIONS

To study the behaviour of transmission line, a transmission can be considered to be made up of a number of identical symmetrical T sections connected in series as in Fig.1.2. If the last section is terminated with its characteristic impedance, the input impedance at the first section is Z_0. Each section is terminated by the input impedance of the following section.

$$\sqrt{\frac{Z_1}{Z_2}\left(1+\frac{Z_1}{4\,Z_2}\right)} = \sqrt{\frac{Z_1}{Z_2}}\left(1+\frac{Z_1}{4\,Z_2}\right)^{\frac{1}{2}}$$

By the binomial theorem,

$$\sqrt{\frac{Z_1}{Z_2}\left(1+\frac{Z_1}{4\,Z_2}\right)} = \sqrt{\frac{Z_1}{Z_2}}\left[1+\frac{1}{2}\left(\frac{Z_1}{4\,Z_2}\right)-\frac{1}{8}\left(\frac{Z_1}{4\,Z_2}\right)^2+\ldots\ldots\right]$$

Substituting this value in e^{γ} equation,

$$e^{\gamma} = 1+\frac{Z_1}{2\,Z_2}+\sqrt{\frac{Z_1}{Z_2}\left(1+\frac{Z_1}{4\,Z_2}\right)}$$

$$= 1+\frac{Z_1}{2\,Z_2}+\sqrt{\frac{Z_1}{Z_2}}+\frac{1}{8}\left(\frac{Z_1}{Z_2}\right)\sqrt{\frac{Z_1}{Z_2}}-\frac{1}{128}\left(\frac{Z_1}{Z_2}\right)^2\sqrt{\frac{Z_1}{Z_2}}+\ldots\ldots$$

$$= 1+\sqrt{\frac{Z_1}{Z_2}}+\frac{1}{2}\left(\sqrt{\frac{Z_1}{Z_2}}\right)^2+\frac{1}{8}\left(\sqrt{\frac{Z_1}{Z_2}}\right)^3-\frac{1}{128}\left(\sqrt{\frac{Z_1}{Z_2}}\right)^5+\ldots..$$

When applied to the incremental length of line Δx, then $Z_1 = Z\,\Delta x$, $Z_2 = \frac{1}{Y\,\Delta x}$ and propagation constant becomes $\gamma\,\Delta x$,

$$e^{\gamma\Delta x} = 1+\sqrt{ZY}\,\Delta x+\frac{1}{2}(\sqrt{ZY})^2(\Delta x)^2+\frac{1}{8}(\sqrt{ZY})^3(\Delta x)^3-128(\sqrt{ZY})^5(\Delta x)^5$$

Series expansion for an exponential $e^{\gamma\Delta x}$ is

$$e^{\gamma\Delta x} = 1+\gamma\Delta x+\frac{\gamma^2(\Delta x)^2}{2!}+\frac{\gamma^3(\Delta x)^3}{3!}+\ldots\ldots$$

Equating the above two expressions,

$$\sqrt{ZY}\,\Delta x+\frac{(\sqrt{ZY})^2(\Delta x)^2}{2}+\frac{(\sqrt{ZY})^2(\Delta x)^3}{8}+\ldots = \gamma\Delta x+\frac{\gamma^2(\Delta x)^2}{2}+\frac{\gamma^3(\Delta x)^3}{6}+\ldots$$

$$\gamma+\frac{\gamma^2\,\Delta x}{2}+\frac{\gamma^3(\Delta x)^2}{6}+\ldots\ldots = \sqrt{ZY}+\frac{(\sqrt{ZY})^2}{2}\Delta x+\frac{(\sqrt{ZY})^3(\Delta x)^2}{8}+\ldots\ldots$$

If Δx tends to zero then,

$$\gamma = \sqrt{ZY}$$

This is the value of propagation constant in terms of Z and Y.

Since each conductor of transmission line has a certain length and diameter, it must have resistance and inductance; moreover the two conductors are separated by a dielectric medium (say, air), therefore there must be a capacitance between them. This dielectric between the conducting wires may not be perfect, and hence a leakage current will flow creating leakage (shunt) capacitance between the conductors. These four parameters resistance (R), inductance (L), capacitance (C) and conductance (G), all distributed along the lines are known as

Consider a T section of transmission line of length dx. Let V + dV be the voltage and I + dI be the current at one end of T section. Let V be the voltage and I be the current at the other end of this section.

The series impedance of a small section dx is $(R + jL\omega)\,dx$. The shunt admittance of this section dx is $(G + jC\omega)\,dx$.

The voltage drop across the series impedance of T sections *i.e.*, the potential difference between the two ends of T section is

$$V + dV - V = I(R + j\omega L)\,dx$$

$$dV = I(R + j\omega L)\,dx$$

$$\frac{dV}{dx} = I(R + j\omega L) \qquad \dots (1.1)$$

$$\frac{dV}{dx} = IZ$$

The current difference between the two ends of T section is due to the voltage drop across the shunt admittance.

$$I + dI - I = V(G + j\omega C)\,dx$$

$$dI = V(G + j\omega C)\,dx$$

$$\frac{dI}{dx} = V(G + j\omega C) \qquad \dots (1.2)$$

$$\frac{dI}{dx} = VY$$

Differentiating equation (1.1) w.r.t. 'x',

$$\frac{d^2V}{dx^2} = (R + j\omega L)\,\frac{dI}{dx}$$

Substituting the value of $\frac{dI}{dx}$ in the above equation

$$\frac{d^2V}{dx^2} = (R + j\omega L)(G + j\omega C)\,V \qquad \dots (1.3)$$

Differentiating equation (1.2) w.r.t. 'x'

$$\frac{d^2I}{dx^2} = (G + j\omega C)\,\frac{dV}{dx}$$

Substituting the value of $\frac{dV}{dx}$ in the above equation

$$\frac{d^2I}{dx^2} = (R + j\omega L)(G + j\omega C)\,I \qquad \dots (1.4)$$

But propagation constant is given by

$$\gamma = \sqrt{(R + j\omega L)(G + j\omega C)} = \sqrt{ZY}$$

Substituting the value of γ in equation (1.3) and (1.4),

$$\text{then} \quad \frac{d^2V}{dx^2} = \gamma^2 V$$

$$\frac{d^2I}{dx^2} = \gamma^2 I$$

The solutions of the above linear differential equations are

$$V = A\, e^{\gamma x} + B\, e^{-\gamma x} \qquad \ldots (1.5)$$

$$I = C\, e^{\gamma x} + D\, e^{-\gamma x} \qquad \ldots (1.6)$$

where A, B, C and D are arbitrary constants.

Differentiating the equation (1.5), w.r.t. 'x'

$$\frac{dV}{dx} = A\gamma\, e^{\gamma x} - B\gamma\, e^{-\gamma x}$$

$$\text{But} \quad \frac{dV}{dx} = IZ$$

$$IZ = A\gamma\, e^{\gamma x} - B\gamma\, e^{-\gamma x}$$

$$= A\sqrt{ZY}\; e^{\sqrt{ZY}\,x} - B\sqrt{ZY}\; e^{-\sqrt{ZY}\,x} \qquad [\because \gamma = \sqrt{ZY}\,]$$

$$I = A\sqrt{\frac{Y}{Z}}\; e^{\sqrt{ZY}\,x} - B\sqrt{\frac{Y}{Z}}\; e^{-\sqrt{ZY}\,x} \qquad \ldots (1.7)$$

Similarly, differentiating the equation (1.6) w.r.t. 'x'

$$\frac{dI}{dx} = C\gamma\, e^{\gamma x} - D\gamma\, e^{-\gamma x}$$

$$\text{But} \quad \frac{dI}{dx} = VY$$

$$VY = C\gamma\, e^{\gamma x} - D\gamma\, e^{-\gamma x}$$

$$= C\sqrt{ZY}\; e^{\sqrt{ZY}\,x} - D\sqrt{ZY}\; e^{-\sqrt{ZY}\,x}$$

$$V = C\sqrt{\frac{Z}{Y}}\; e^{\sqrt{ZY}\,x} - D\sqrt{\frac{Z}{Y}}\; e^{-\sqrt{ZY}\,x} \qquad \ldots (1.8)$$

Since the distance x is measured from the receiving end of the transmission line,

$$x = 0, \qquad \therefore \quad I = I_R$$

$$V = V_R$$

$$V_R = I_R Z_R$$

where I_R is the current in the receiving end of line

V_R is the voltage across the receiving end of the lines

Z_R is the impedance of receiving end

Substituting this condition in equations (1.5), (1.6), (1.7) and (1.8).

$$V_R = A + B \qquad \text{... (1.9)}$$

$$I_R = C + D \qquad \text{... (1.10)}$$

$$I_R = A\sqrt{\frac{Y}{Z}} - B\sqrt{\frac{Y}{Z}} \qquad \text{... (1.11)}$$

$$V_R = C\sqrt{\frac{Z}{Y}} - D\sqrt{\frac{Z}{Y}} \qquad \text{... (1.12)}$$

To solve these equations,

$$\text{Let } x = \sqrt{\frac{Z}{Y}} \quad \text{and} \quad \frac{1}{x} = \sqrt{\frac{Y}{Z}}$$

$$\text{Then } I_R = \frac{A}{x} - \frac{B}{x}$$

$$= \frac{1}{x}(A - B)$$

$$\text{But } I_R = C + D$$

$$C + D = \frac{1}{x}(A - B)$$

$$Cx + Dx = A - B$$

$$A - B = Cx + Dx \qquad \text{... (1.13)}$$

Similarly, equation (1.12) becomes,

$$V_R = Cx - Dx$$

$$\text{But } V_R = A + B$$

$$A + B = Cx - Dx \qquad \text{... (1.14)}$$

$$A - B = Cx + Dx \qquad \text{... (1.13)}$$

Adding the equations (1.13) and (1.14),

$$2A = 2Cx$$

$$A = Cx$$

Similarly subtracting the equations (1.13) and (1.14),

$$2B = -2x\,Dx$$

$$B = -Dx$$

Substituting the values of A and B in the following equations.

$$V_R = A + B$$
$$= Cx - Dx$$
$$\text{But } I_R = C + D$$
$$I_R x = Cx + Dx \qquad \dots (1.15)$$
$$V_R = Cx - Dx \qquad \dots (1.16)$$

Adding the equations (1.15) and (1.16),

$$2Cx = I_R x + V_R$$
$$C = \frac{I_R}{2} + \frac{V_R}{2x}$$
$$\boxed{\therefore C = \frac{I_R}{2} + \frac{V_R}{2}\sqrt{\frac{Y}{Z}}} \qquad \dots (1.17) \qquad \left[\because x = \sqrt{\frac{Z}{Y}}\ \right]$$

Subtracting the equations (1.15) and (1.16),

$$2Dx = I_R x - V_R$$
$$D = \frac{I_R}{2} - \frac{V_R}{2x}$$
$$\boxed{\therefore D = \frac{I_R}{2} - \frac{V_R}{2}\sqrt{\frac{Y}{Z}}} \qquad \dots (1.18)$$
$$\text{But } A = Cx$$
$$A = \frac{I_R}{2}x + \frac{V_R}{2}$$
$$\boxed{\therefore A = \frac{V_R}{2} + \frac{I_R}{2}\sqrt{\frac{Z}{Y}}} \qquad \dots (1.19)$$
$$B = -Dx$$
$$B = -\frac{I_R}{2}x + \frac{V_R}{2}$$
$$\boxed{\therefore B = \frac{V_R}{2} - \frac{I_R}{2}\sqrt{\frac{Z}{Y}}} \qquad \dots (1.20)$$

The characteristic impedance is defined as

$$Z_o = \sqrt{\frac{Z}{Y}}$$
$$= \sqrt{\frac{R + j\omega L}{G + j\omega C}} \qquad \dots (1.21)$$

Substituting the value of Z_0 in equations (1.19), (1.20), (1.17) and (1.18),

$$A = \frac{V_R}{2} + \frac{I_R}{2}\sqrt{\frac{Z}{Y}}$$

$$A = \frac{V_R}{2} + \frac{V_R}{2\,Z_R}\,Z_o$$

$$\boxed{A = \frac{V_R}{2}\left[1 + \frac{Z_o}{Z_R}\right]} \qquad \ldots (1.22)$$

$$B = \frac{V_R}{2} - \frac{I_R}{2}\sqrt{\frac{Z}{Y}}$$

$$= \frac{V_R}{2} - \frac{V_R}{2\,Z_R}\,Z_o$$

$$\boxed{B = \frac{V_R}{2}\left[1 - \frac{Z_o}{Z_R}\right]} \qquad \ldots (1.23)$$

$$C = \frac{I_R}{2} + \frac{V_R}{2}\sqrt{\frac{Y}{Z}}$$

$$= \frac{I_R}{2} + \frac{I_R\,Z_R}{2\,Z_o} \qquad [\because V_R = I_R\,Z_R]$$

$$\boxed{C = \frac{I_R}{2}\left[1 + \frac{Z_R}{Z_o}\right]} \qquad \ldots (1.24)$$

$$D = \frac{I_R}{2} - \frac{V_R}{2}\sqrt{\frac{Y}{Z}}$$

$$= \frac{I_R}{2} - \frac{I_R\,Z_R}{2\,Z_o}$$

$$\boxed{D = \frac{I_R}{2}\left[1 + \frac{Z_R}{Z_o}\right]} \qquad \ldots (1.25)$$

Substituting the values of A, B, C and D in equations (1.5) and (1.6), the solutions of the differential equations are

$$V = \frac{V_R}{2}\left(1 + \frac{Z_o}{Z_R}\right) e^{\sqrt{ZY}\,x} + \frac{V_R}{2}\left(1 - \frac{Z_o}{Z_R}\right) e^{-\sqrt{ZY}\,x} \qquad \ldots (1.26)$$

$$I = \frac{I_R}{2}\left(1 + \frac{Z_R}{Z_o}\right) e^{\sqrt{ZY}\,x} + \frac{I_R}{2}\left(1 - \frac{Z_R}{Z_o}\right) e^{-\sqrt{ZY}\,x} \qquad \ldots (1.27)$$

$$V = \frac{V_R}{2}\left[\left(1 + \frac{Z_o}{Z_R}\right) e^{\sqrt{ZY}\,x} + \left(1 - \frac{Z_o}{Z_R}\right) e^{-\sqrt{ZY}\,x}\right] \qquad \ldots (1.28)$$

$$I = \frac{I_R}{2}\left[\left(1+\frac{Z_R}{Z_o}\right)e^{\sqrt{ZY}\,x} + \left(1-\frac{Z_R}{Z_o}\right)e^{-\sqrt{ZY}\,x}\right] \quad \ldots (1.29)$$

After simplification,

$$V = \frac{V_R}{2}e^{\sqrt{ZY}\,x} + \frac{V_R}{2}\frac{Z_o}{Z_R}e^{\sqrt{ZY}\,x} + \frac{V_R}{2}e^{-\sqrt{ZY}\,x} - \frac{V_R}{2}\frac{Z_o}{Z_R}e^{-\sqrt{ZY}\,x}$$

$$I = \frac{I_R}{2}e^{\sqrt{ZY}\,x} + \frac{I_R}{2}\frac{Z_R}{Z_o}e^{\sqrt{ZY}\,x} + \frac{I_R}{2}e^{-\sqrt{ZY}\,x} - \frac{I_R}{2}\frac{Z_R}{Z_o}e^{-\sqrt{ZY}\,x}$$

$$V = V_R\left(\frac{e^{\sqrt{ZY}\,x} + e^{-\sqrt{ZY}\,x}}{2}\right) + I_R Z_o\left(\frac{e^{\sqrt{ZY}\,x} - e^{-\sqrt{ZY}\,x}}{2}\right) \quad [\because V_R = I_R Z_R]$$

$$I = I_R\left(\frac{e^{\sqrt{ZY}\,x} + e^{-\sqrt{ZY}\,x}}{2}\right) + \frac{V_R}{Z_o}\,(e^{\sqrt{ZY}\,x} - e^{-\sqrt{ZY}\,x}) \quad \left[\because I_R = \frac{V_R}{Z_R}\right]$$

Then equations can be written in terms of hyperbolic functions.

$$\boxed{\begin{aligned} V &= V_R \cosh\sqrt{ZY}\;x + I_R Z_o \sinh\sqrt{ZY}\;x \\ I &= I_R \cosh\sqrt{ZY}\;x + \frac{V_R}{Z_o}\sinh\sqrt{ZY}\;x \end{aligned}} \quad \begin{aligned} &\ldots (1.30) \\ &\ldots (1.31) \end{aligned}$$

These are the equations for voltage and current of a transmission line at any distance 'x' from the receiving end of transmission line.

The equations for voltage and current at the sending send of a transmission line of length 'l' are given by

$$V_S = V_R \cosh\sqrt{ZY}\,l + \frac{V_R}{Z_R}Z_o \sinh\sqrt{ZY}\,l \quad \left[\because I_R = \frac{V_R}{Z_R}\right]$$

$$I_S = I_R \cosh\sqrt{ZY}\,l + \frac{I_R Z_R}{Z_o}\sinh\sqrt{ZY}\;l \quad [\because V_R = I_R Z_R]$$

$$V_S = V_R\left[\cos\sqrt{ZY}\;l + \frac{Z_0}{Z_R}\sinh\sqrt{ZY}\;l\right] \quad \ldots (1.32)$$

$$I_S = I_R\left[\cos\sqrt{ZY}\;l + \frac{Z_R}{Z_0}\sinh\sqrt{ZY}\;l\right] \quad \ldots (1.33)$$

1.4. WAVELENGTH AND VELOCITY OF PROPAGATION

The propagation constant (γ) and characteristic impedance (Z_0) are called secondary constants of a transmission line.

Propagation constant is usually a complex quantity.

$$\gamma = \alpha + j\beta$$

where α is the attenuation constant.

β is the phase shift.

$$\gamma = \sqrt{ZY}$$

where $Z = R + j\omega L$

$$Y = G + j\omega C$$

The characteristic impedance of the transmission line is also a complex quantity.

$$Z_0 = \sqrt{\frac{Z}{Y}}$$

$$Z_0 = \sqrt{\frac{R + j\omega L}{G + j\omega C}} \qquad \ldots (1.34)$$

Propagation constant is $\gamma = \alpha + i\beta$

$$= \sqrt{(R + j\omega L)(G + j\omega C)}$$

$$\alpha + i\beta = \sqrt{RG - \omega^2 LC + j\omega(LG + RC)} \qquad \ldots (1.35)$$

Squaring on both sides,

$$(\alpha + j\beta)^2 = RG - \omega^2 LC + j\omega(LG + RC)$$

$$\alpha^2 - \beta^2 + 2j\,\alpha\beta = RG - \omega^2 LC + j\omega\,(LG + RC) \qquad \ldots (1.36)$$

Equating real parts,

$$\alpha^2 - \beta^2 = RG - \omega^2 LC$$

$$\alpha^2 = \beta^2 + RG - \omega^2 LC \qquad \ldots (1.37)$$

Equating imaginary parts,

$$2\,\alpha\beta = \omega\,(LG + RC)$$

Squaring on both sides,

$$4\,\alpha^2\,\beta^2 = \omega^2\,(LG + RC)^2$$

$$\alpha^2\,\beta^2 = \frac{\omega^2}{4}\,(LG + RC)^2$$

Substituting the value of α^2 [eqn. (1.37)] in the above equation,

$$(\beta^2 + RG - \omega^2 LC)\,\beta^2 = \frac{\omega^2}{4}\,(LG + RC)^2$$

$$\beta^4 + \beta^2\,(RG - \omega^2 LC) - \frac{\omega^2}{4}\,(LG + RC)^2 = 0$$

The solution of the quadratic equation is

$$\beta^2 = \frac{-(RG - \omega^2 LC) \pm \sqrt{(RG - \omega^2 LC)^2 + \omega^2\,(LG + RC)^2}}{2}$$

By neglecting the negative values,

$$\therefore \quad \beta = \sqrt{\frac{\omega^2 LC - RG + \sqrt{(RG - \omega^2 LC)^2 + \omega^2 (LG + RC)^2}}{2}} \quad \dots (1.38)$$

$$\alpha^2 = \beta^2 + RG - \omega^2 LC \quad \dots (1.37)$$

Substituting the value of β [eqn. (1.38)] in the above equation,

$$\alpha^2 = \frac{\omega^2 LC - RG + \sqrt{(RG - \omega^2 LC)^2 + \omega^2 (LG + RC)^2}}{2} + RG - \omega^2 LC$$

$$= \frac{RG - \omega^2 LC + \sqrt{(RG - \omega^2 LC)^2 + \omega^2 (LG + RC)^2}}{2}$$

$$\therefore \quad \alpha = \sqrt{\frac{RG - \omega^2 LC + \sqrt{(RG - \omega^2 LC)^2 + \omega^2 (LG + RC)^2}}{2}} \quad \dots (1.39)$$

For a perfect transmission line R = 0 and G = 0,

$$\beta^2 = \omega^2 LC$$

$$\therefore \quad \beta = \omega\sqrt{LC} \quad \text{[only positive value]}$$

Velocity :

The velocity of propagation is given by,

$$v = \lambda f$$

$$= 2\pi f \frac{\lambda}{2\pi}$$

$$v = \frac{\omega}{\beta} \quad \left[\because \beta = \frac{2\pi}{\lambda} \text{ and } \omega = 2\pi f\right]$$

Substituting the value of $\beta = \omega\sqrt{LC}$

$$\therefore \quad v = \frac{\omega}{\omega\sqrt{LC}}$$

$$v = \frac{1}{\sqrt{LC}}$$

This is the velocity of propagation for an ideal line.

Wavelength :

The distance travelled by the wave along the line while the phase angle is changing through 2π radians is called wavelength.

$$\beta\lambda = 2\pi$$

$$\lambda = \frac{2\pi}{\beta} \quad \text{or} \quad \lambda = \frac{v}{f}$$

1.5. INPUT IMPEDANCE AND TRANSFER IMPEDANCE OF TRANSMISSION LINE

Input impedance :

The equations for voltage and current at the sending end of a transmission line of length 'l' are given by

E
$$V_S = V_R\left(\cosh\sqrt{ZY}\,l + \frac{Z_0}{Z_R}\sinh\sqrt{ZY}\,l\right) \qquad \ldots (1.32)$$

I
$$I_S = I_R\left(\cosh\sqrt{ZY}\,l + \frac{Z_R}{Z_0}\sinh\sqrt{ZY}\,l\right) \qquad \ldots (1.33)$$

The input impedance of the transmission line is,

$$Z_S = \frac{V_S}{I_S}$$

$$= \frac{V_R\left(\cosh\sqrt{ZY}\,l + \frac{Z_0}{Z_R}\sinh\sqrt{ZY}\,l\right)}{I_R\left(\cosh\sqrt{ZY}\,l + \frac{Z_R}{Z_0}\sinh\sqrt{ZY}\,l\right)}$$

$$= \frac{I_R Z_R\left(\cosh\sqrt{ZY}\,l + \frac{Z_0}{Z_R}\sinh\sqrt{ZY}\,l\right)}{I_R\left(\cosh\sqrt{ZY}\,l + \frac{Z_R}{Z_0}\sinh\sqrt{ZY}\,l\right)}$$

$$Z_S = \frac{Z_0\,(Z_R\cosh\sqrt{ZY}\,l + Z_0\sinh\sqrt{ZY}\,l)}{(Z_0\cosh\sqrt{ZY}\,l + Z_R\sinh\sqrt{ZY}\,l)} \qquad \ldots (1.40)$$

Let $\sqrt{ZY} = \gamma$

The input impedance of the line is

$$Z_S = Z_0\left[\frac{Z_R\cosh\gamma l + Z_0\sinh\gamma l}{Z_0\cosh\gamma l + Z_R\sinh\gamma l}\right]$$

or $$Z_S = Z_0\left[\frac{Z_R + Z_0\tanh\gamma l}{Z_0 + Z_R\tanh\gamma l}\right]$$

In a different form, the equations for voltage and current at transmitting end of a line is given by equations (1.28) and (1.29),

$$V_S = \frac{V_R}{2}\left[\left(1 + \frac{Z_0}{Z_R}\right)e^{\sqrt{ZY}\,l} + \left(1 - \frac{Z_0}{Z_R}\right)e^{-\sqrt{ZY}\,l}\right] \qquad \ldots (1.28)$$

$$I_S = \frac{I_R}{2}\left[\left(1 + \frac{Z_R}{Z_0}\right)e^{\sqrt{ZY}\,l} + \left(1 - \frac{Z_R}{Z_0}\right)e^{-\sqrt{ZY}\,l}\right] \qquad \ldots (1.29)$$

or $$V_S = \frac{V_R}{2}\left[\left(\frac{Z_R+Z_0}{Z_R}\right) e^{\sqrt{ZY}\,l} + \left(\frac{Z_R-Z_0}{Z_R}\right) e^{-\sqrt{ZY}\,l}\right]$$

$$I_S = \frac{I_R}{2}\left[\left(\frac{Z_R+Z_0}{Z_0}\right) e^{\sqrt{ZY}\,l} + \left(\frac{Z_0-Z_R}{Z_0}\right) e^{-\sqrt{ZY}\,l}\right]$$

or $$V_S = \left(\frac{V_R}{2}\right)\left(\frac{Z_R+Z_0}{Z_R}\right)\left[e^{\sqrt{ZY}\,l} + \left(\frac{Z_R-Z_0}{Z_R+Z_0}\right) e^{-\sqrt{ZY}\,l}\right] \qquad \ldots (1.41)$$

$$I_S = \frac{I_R}{2}\left(\frac{Z_R+Z_0}{Z_0}\right)\left[e^{\sqrt{ZY}\,l} - \left(\frac{Z_R-Z_0}{Z_R+Z_0}\right) e^{-\sqrt{ZY}\,l}\right] \qquad \ldots (1.42)$$

The input impedance of the transmission line is given by,

$$Z_S = \frac{V_S}{I_S} = Z_0\left[\frac{e^{\sqrt{ZY}\,l} + \left(\frac{Z_R-Z_0}{Z_R+Z_0}\right) e^{-\sqrt{ZY}\,l}}{e^{\sqrt{ZY}\,l} - \left(\frac{Z_R-Z_0}{Z_R+Z_0}\right) e^{-\sqrt{ZY}\,l}}\right] \quad [\because V_R = I_R Z_R] \qquad \ldots (1.43)$$

Let $\sqrt{ZY} = \gamma$

The input impedance of the transmission line is,

$$Z_S = Z_0\left[\frac{e^{\gamma l} + \left(\frac{Z_R-Z_0}{Z_R+Z_0}\right) e^{-\gamma l}}{e^{\gamma l} - \left(\frac{Z_R-Z_0}{Z_R+Z_0}\right) e^{-\gamma l}}\right] \qquad \ldots (1.44)$$

If the line is terminated with its characteristic impedance *i.e.*, $Z_R = Z_0$, then the input impedance becomes equal to its characteristic impedance.

$$Z_S = Z_0$$

The input impedance of an infinite line is determined by letting $l \to \infty$.

$$\therefore\ Z_S = Z_0$$

It is found that a line of finite length, terminated with its characteristic impedance, appears to the transmitting end generator as an infinite line. A finite line terminated with Z_0 and an infinite line are same by measurements at the source.

If $K = \frac{Z_R - Z_0}{Z_R + Z_0}$, then

$$Z_S = Z_0\left[\frac{e^{\gamma l} + K e^{-\gamma l}}{e^{\gamma l} - K e^{-\gamma l}}\right] \qquad \ldots (1.45)$$

Transfer impedance :

Transfer impedance is used to determine the current at the receiving end if voltage at transmitting end is known. Transfer impedance of a transmission line is defined as the ratio of voltage at the sending end (transmitted voltage) to the current at the receiving end (received current).

$$Z_T = \frac{V_S}{I_R}$$

Equation (1.41) becomes

$$V_S = \frac{V_R (Z_R + Z_0)}{2 Z_R} \left(e^{\gamma l} + K e^{-\gamma l}\right)$$

$$V_S = \frac{I_R (Z_R + Z_0)}{2} \left(e^{\gamma l} + K e^{-\gamma l}\right) \qquad [\because V_R = I_R Z_R]$$

$$Z_T = \frac{V_S}{I_R} = \frac{Z_R + Z_0}{2} \left(e^{\gamma l} + K e^{-\gamma l}\right)$$

$$= \frac{Z_R + Z_0}{2} \left(e^{\gamma l} + \frac{Z_R - Z_0}{Z_R + Z_0} e^{-\gamma l}\right)$$

$$= \left(\frac{Z_R + Z_0}{2}\right) e^{\gamma l} + \left(\frac{Z_R - Z_0}{2}\right) e^{-\gamma l}$$

$$= Z_R \left(\frac{e^{\gamma l} + e^{-\gamma l}}{2}\right) + Z_0 \left(\frac{e^{\gamma l} - e^{-\gamma l}}{2}\right)$$

$$= Z_R \cosh \gamma l + Z_0 \sinh \gamma l$$

$$\left[\because \frac{e^{\gamma l} + e^{-\gamma l}}{2} = \cosh \gamma l \quad \text{and} \quad \frac{e^{\gamma l} - e^{-\gamma l}}{2} = \sinh \gamma l\right]$$

$$Z_T = Z_R \cosh \gamma l + Z_0 \sinh \gamma l$$

1.6. LINE DISTORTION

Signal (*e.g.*, voice) transmitted over a transmission line is normally complex and consists of many frequency components. Such voice voltage will not have all frequencies transmitted with equal attenuation and equal time delay, the received waveform will not be identical with the input waveform at the sending end. This variation is known as ***distortion***. There are two types of line distortions. They are frequency distortion and delay distortion.

Frequency Distortion : A complex (voice) voltage transmitted on a transmission line will not be attenuated equally and the received waveform will not be identical with the input waveform at the transmitting end. This variation is known as frequency distortion.

The attenuation constant is given by

Consider a T section of transmission line of length dx. Let V + dV be the voltage and I + dI be the current at one end of T section. Let V be the voltage and I be the current at the other end of this section.

The series impedance of a small section dx is $(R + jL\omega)\, dx$. The shunt admittance of this section dx is $(G + jC\omega)\, dx$.

The voltage drop across the series impedance of T sections *i.e.*, the potential difference between the two ends of T section is

$$V + dV - V = I(R + j\omega L)\, dx$$

$$dV = I(R + j\omega L)\, dx$$

$$\frac{dV}{dx} = I(R + j\omega L) \qquad \dots (1.1)$$

$$\frac{dV}{dx} = IZ$$

The current difference between the two ends of T section is due to the voltage drop across the shunt admittance.

$$I + dI - I = V(G + j\omega C)\, dx$$

$$dI = V(G + j\omega C)\, dx$$

$$\frac{dI}{dx} = V(G + j\omega C) \qquad \dots (1.2)$$

$$\frac{dI}{dx} = VY$$

Differentiating equation (1.1) w.r.t. 'x',

$$\frac{d^2V}{dx^2} = (R + j\omega L)\,\frac{dI}{dx}$$

Substituting the value of $\frac{dI}{dx}$ in the above equation

$$\frac{d^2V}{dx^2} = (R + j\omega L)(G + j\omega C)\,V \qquad \dots (1.3)$$

Differentiating equation (1.2) w.r.t. 'x'

$$\frac{d^2I}{dx^2} = (G + j\omega C)\,\frac{dV}{dx}$$

Substituting the value of $\frac{dV}{dx}$ in the above equation

$$\frac{d^2I}{dx^2} = (R + j\omega L)(G + j\omega C)\,I \qquad \dots (1.4)$$

But propagation constant is given by

$$\gamma = \sqrt{(R + j\omega L)(G + j\omega C)} = \sqrt{ZY}$$

Substituting the value of γ in equation (1.3) and (1.4),

$$\text{then} \quad \frac{d^2V}{dx^2} = \gamma^2 V$$

$$\frac{d^2I}{dx^2} = \gamma^2 I$$

The solutions of the above linear differential equations are

$$V = A\, e^{\gamma x} + B\, e^{-\gamma x} \qquad \dots (1.5)$$

$$I = C\, e^{\gamma x} + D\, e^{-\gamma x} \qquad \dots (1.6)$$

where A, B, C and D are arbitrary constants.

Differentiating the equation (1.5), w.r.t. 'x'

$$\frac{dV}{dx} = A\gamma\, e^{\gamma x} - B\gamma\, e^{-\gamma x}$$

$$\text{But} \quad \frac{dV}{dx} = IZ$$

$$IZ = A\gamma\, e^{\gamma x} - B\gamma\, e^{-\gamma x}$$

$$= A\sqrt{ZY}\; e^{\sqrt{ZY}\,x} - B\sqrt{ZY}\; e^{-\sqrt{ZY}\,x} \qquad [\because \gamma = \sqrt{ZY}\,]$$

$$I = A\sqrt{\frac{Y}{Z}}\; e^{\sqrt{ZY}\,x} - B\sqrt{\frac{Y}{Z}}\; e^{-\sqrt{ZY}\,x} \qquad \dots (1.7)$$

Similarly, differentiating the equation (1.6) w.r.t. 'x'

$$\frac{dI}{dx} = C\gamma\, e^{\gamma x} - D\gamma\, e^{-\gamma x}$$

$$\text{But} \quad \frac{dI}{dx} = VY$$

$$VY = C\gamma\, e^{\gamma x} - D\gamma\, e^{-\gamma x}$$

$$= C\sqrt{ZY}\; e^{\sqrt{ZY}\,x} - D\sqrt{ZY}\; e^{-\sqrt{ZY}\,x}$$

$$V = C\sqrt{\frac{Z}{Y}}\; e^{\sqrt{ZY}\,x} - D\sqrt{\frac{Z}{Y}}\; e^{-\sqrt{ZY}\,x} \qquad \dots (1.8)$$

Since the distance x is measured from the receiving end of the transmission line,

$$x = 0, \qquad \therefore \quad I = I_R$$

$$V = V_R$$

$$V_R = I_R Z_R$$

where I_R is the current in the receiving end of line

V_R is the voltage across the receiving end of the lines

Z_R is the impedance of receiving end

Substituting this condition in equations (1.5), (1.6), (1.7) and (1.8).

$$V_R = A + B \quad \dots (1.9)$$

$$I_R = C + D \quad \dots (1.10)$$

$$I_R = A\sqrt{\frac{Y}{Z}} - B\sqrt{\frac{Y}{Z}} \quad \dots (1.11)$$

$$V_R = C\sqrt{\frac{Z}{Y}} - D\sqrt{\frac{Z}{Y}} \quad \dots (1.12)$$

To solve these equations,

$$\text{Let } x = \sqrt{\frac{Z}{Y}} \quad \text{and} \quad \frac{1}{x} = \sqrt{\frac{Y}{Z}}$$

$$\text{Then } I_R = \frac{A}{x} - \frac{B}{x}$$

$$= \frac{1}{x}(A - B)$$

$$\text{But } I_R = C + D$$

$$C + D = \frac{1}{x}(A - B)$$

$$Cx + Dx = A - B$$

$$A - B = Cx + Dx \quad \dots (1.13)$$

Similarly, equation (1.12) becomes,

$$V_R = Cx - Dx$$

$$\text{But } V_R = A + B$$

$$A + B = Cx - Dx \quad \dots (1.14)$$

$$A - B = Cx + Dx \quad \dots (1.13)$$

Adding the equations (1.13) and (1.14),

$$2A = 2Cx$$

$$A = Cx$$

Similarly subtracting the equations (1.13) and (1.14),

$$2B = -2x\,Dx$$

$$B = -Dx$$

Substituting the values of A and B in the following equations.

$$V_R = A + B$$

$$= Cx - Dx$$

$$\text{But } I_R = C + D$$

$$I_R x = Cx + Dx \qquad \dots (1.15)$$

$$V_R = Cx - Dx \qquad \dots (1.16)$$

Adding the equations (1.15) and (1.16),

$$2Cx = I_R x + V_R$$

$$C = \frac{I_R}{2} + \frac{V_R}{2x}$$

$$\boxed{\therefore C = \frac{I_R}{2} + \frac{V_R}{2}\sqrt{\frac{Y}{Z}}} \qquad \dots (1.17) \qquad \left[\because x = \sqrt{\frac{Z}{Y}}\right]$$

Subtracting the equations (1.15) and (1.16),

$$2Dx = I_R x - V_R$$

$$D = \frac{I_R}{2} - \frac{V_R}{2x}$$

$$\boxed{\therefore D = \frac{I_R}{2} - \frac{V_R}{2}\sqrt{\frac{Y}{Z}}} \qquad \dots (1.18)$$

$$\text{But } A = Cx$$

$$A = \frac{I_R}{2}x + \frac{V_R}{2}$$

$$\boxed{\therefore A = \frac{V_R}{2} + \frac{I_R}{2}\sqrt{\frac{Z}{Y}}} \qquad \dots (1.19)$$

$$B = -Dx$$

$$B = -\frac{I_R}{2}x + \frac{V_R}{2}$$

$$\boxed{\therefore B = \frac{V_R}{2} - \frac{I_R}{2}\sqrt{\frac{Z}{Y}}} \qquad \dots (1.20)$$

The characteristic impedance is defined as

$$Z_o = \sqrt{\frac{Z}{Y}}$$

$$= \sqrt{\frac{R + j\omega L}{G + j\omega C}} \qquad \dots (1.21)$$

Substituting the value of Z_0 in equations (1.19), (1.20), (1.17) and (1.18),

$$A = \frac{V_R}{2} + \frac{I_R}{2}\sqrt{\frac{Z}{Y}}$$

$$A = \frac{V_R}{2} + \frac{V_R}{2\,Z_R}\,Z_o$$

$$\boxed{A = \frac{V_R}{2}\left[1 + \frac{Z_o}{Z_R}\right]} \qquad \dots (1.22)$$

$$B = \frac{V_R}{2} - \frac{I_R}{2}\sqrt{\frac{Z}{Y}}$$

$$= \frac{V_R}{2} - \frac{V_R}{2\,Z_R}\,Z_o$$

$$\boxed{B = \frac{V_R}{2}\left[1 - \frac{Z_o}{Z_R}\right]} \qquad \dots (1.23)$$

$$C = \frac{I_R}{2} + \frac{V_R}{2}\sqrt{\frac{Y}{Z}}$$

$$= \frac{I_R}{2} + \frac{I_R\,Z_R}{2\,Z_o} \qquad [\because V_R = I_R\,Z_R]$$

$$\boxed{C = \frac{I_R}{2}\left[1 + \frac{Z_R}{Z_o}\right]} \qquad \dots (1.24)$$

$$D = \frac{I_R}{2} - \frac{V_R}{2}\sqrt{\frac{Y}{Z}}$$

$$= \frac{I_R}{2} - \frac{I_R\,Z_R}{2\,Z_o}$$

$$\boxed{D = \frac{I_R}{2}\left[1 + \frac{Z_R}{Z_o}\right]} \qquad \dots (1.25)$$

Substituting the values of A, B, C and D in equations (1.5) and (1.6), the solutions of the differential equations are

$$V = \frac{V_R}{2}\left(1 + \frac{Z_o}{Z_R}\right)e^{\sqrt{ZY}\,x} + \frac{V_R}{2}\left(1 - \frac{Z_o}{Z_R}\right)e^{-\sqrt{ZY}\,x} \qquad \dots (1.26)$$

$$I = \frac{I_R}{2}\left(1 + \frac{Z_R}{Z_o}\right)e^{\sqrt{ZY}\,x} + \frac{I_R}{2}\left(1 - \frac{Z_R}{Z_o}\right)e^{-\sqrt{ZY}\,x} \qquad \dots (1.27)$$

$$V = \frac{V_R}{2}\left[\left(1 + \frac{Z_o}{Z_R}\right)e^{\sqrt{ZY}\,x} + \left(1 - \frac{Z_o}{Z_R}\right)e^{-\sqrt{ZY}\,x}\right] \qquad \dots (1.28)$$

$$I = \frac{I_R}{2}\left[\left(1+\frac{Z_R}{Z_o}\right)e^{\sqrt{ZY}\,x}+\left(1-\frac{Z_R}{Z_o}\right)e^{-\sqrt{ZY}\,x}\right] \quad \ldots (1.29)$$

After simplification,

$$V = \frac{V_R}{2}e^{\sqrt{ZY}\,x}+\frac{V_R}{2}\frac{Z_o}{Z_R}e^{\sqrt{ZY}\,x}+\frac{V_R}{2}e^{-\sqrt{ZY}\,x}-\frac{V_R}{2}\frac{Z_o}{Z_R}e^{-\sqrt{ZY}\,x}$$

$$I = \frac{I_R}{2}e^{\sqrt{ZY}\,x}+\frac{I_R}{2}\frac{Z_R}{Z_o}e^{\sqrt{ZY}\,x}+\frac{I_R}{2}e^{-\sqrt{ZY}\,x}-\frac{I_R}{2}\frac{Z_R}{Z_o}e^{-\sqrt{ZY}\,x}$$

$$V = V_R\left(\frac{e^{\sqrt{ZY}\,x}+e^{-\sqrt{ZY}\,x}}{2}\right)+I_R Z_o\left(\frac{e^{\sqrt{ZY}\,x}-e^{-\sqrt{ZY}\,x}}{2}\right) \quad [\because V_R = I_R Z_R]$$

$$I = I_R\left(\frac{e^{\sqrt{ZY}\,x}+e^{-\sqrt{ZY}\,x}}{2}\right)+\frac{V_R}{Z_o}\,(e^{\sqrt{ZY}\,x}-e^{-\sqrt{ZY}\,x}) \quad \left[\because I_R = \frac{V_R}{Z_R}\right]$$

Then equations can be written in terms of hyperbolic functions.

$$\boxed{\begin{aligned} V &= V_R\cosh\sqrt{ZY}\;x + I_R Z_o \sinh\sqrt{ZY}\;x \\ I &= I_R\cosh\sqrt{ZY}\;x + \frac{V_R}{Z_o}\sinh\sqrt{ZY}\;x \end{aligned}} \quad \begin{aligned} \ldots (1.30) \\ \ldots (1.31) \end{aligned}$$

These are the equations for voltage and current of a transmission line at any distance 'x' from the receiving end of transmission line.

The equations for voltage and current at the sending send of a transmission line of length 'l' are given by

$$V_S = V_R\cosh\sqrt{ZY}\,l + \frac{V_R}{Z_R}Z_o\sinh\sqrt{ZY}\,l \quad \left[\because I_R = \frac{V_R}{Z_R}\right]$$

$$I_S = I_R\cosh\sqrt{ZY}\,l + \frac{I_R Z_R}{Z_o}\sinh\sqrt{ZY}\,l \quad [\because V_R = I_R Z_R]$$

$$V_S = V_R\left[\cos\sqrt{ZY}\,l + \frac{Z_0}{Z_R}\sinh\sqrt{ZY}\,l\right] \quad \ldots (1.32)$$

$$I_S = I_R\left[\cos\sqrt{ZY}\,l + \frac{Z_R}{Z_0}\sinh\sqrt{ZY}\,l\right] \quad \ldots (1.33)$$

1.4. WAVELENGTH AND VELOCITY OF PROPAGATION

The propagation constant (γ) and characteristic impedance (Z_0) are called secondary constants of a transmission line.

Propagation constant is usually a complex quantity.

$$\gamma = \alpha + j\beta$$

where α is the attenuation constant.

β is the phase shift.

$$\gamma = \sqrt{ZY}$$

where $Z = R + j\omega L$

$$Y = G + j\omega C$$

The characteristic impedance of the transmission line is also a complex quantity.

$$Z_0 = \sqrt{\frac{Z}{Y}}$$

$$Z_0 = \sqrt{\frac{R + j\omega L}{G + j\omega C}} \quad \ldots (1.34)$$

Propagation constant is $\gamma = \alpha + i\beta$

$$= \sqrt{(R + j\omega L)(G + j\omega C)}$$

$$\alpha + i\beta = \sqrt{RG - \omega^2 LC + j\omega(LG + RC)} \quad \ldots (1.35)$$

Squaring on both sides,

$$(\alpha + j\beta)^2 = RG - \omega^2 LC + j\omega(LG + RC)$$

$$\alpha^2 - \beta^2 + 2j\,\alpha\beta = RG - \omega^2 LC + j\omega\,(LG + RC) \quad \ldots (1.36)$$

Equating real parts,

$$\alpha^2 - \beta^2 = RG - \omega^2 LC$$

$$\alpha^2 = \beta^2 + RG - \omega^2 LC \quad \ldots (1.37)$$

Equating imaginary parts,

$$2\,\alpha\beta = \omega\,(LG + RC)$$

Squaring on both sides,

$$4\,\alpha^2\beta^2 = \omega^2\,(LG + RC)^2$$

$$\alpha^2\beta^2 = \frac{\omega^2}{4}\,(LG + RC)^2$$

Substituting the value of α^2 [eqn. (1.37)] in the above equation,

$$(\beta^2 + RG - \omega^2 LC)\,\beta^2 = \frac{\omega^2}{4}\,(LG + RC)^2$$

$$\beta^4 + \beta^2\,(RG - \omega^2 LC) - \frac{\omega^2}{4}\,(LG + RC)^2 = 0$$

The solution of the quadratic equation is

$$\beta^2 = \frac{-(RG - \omega^2 LC) \pm \sqrt{(RG - \omega^2 LC)^2 + \omega^2\,(LG + RC)^2}}{2}$$

By neglecting the negative values,

$$\therefore \quad \beta = \sqrt{\frac{\omega^2 LC - RG + \sqrt{(RG - \omega^2 LC)^2 + \omega^2 (LG + RC)^2}}{2}} \qquad \ldots (1.38)$$

$$\alpha^2 = \beta^2 + RG - \omega^2 LC \qquad \ldots (1.37)$$

Substituting the value of β [eqn. (1.38)] in the above equation,

$$\alpha^2 = \frac{\omega^2 LC - RG + \sqrt{(RG - \omega^2 LC)^2 + \omega^2 (LG + RC)^2}}{2} + RG - \omega^2 LC$$

$$= \frac{RG - \omega^2 LC + \sqrt{(RG - \omega^2 LC)^2 + \omega^2 (LG + RC)^2}}{2}$$

$$\therefore \quad \alpha = \sqrt{\frac{RG - \omega^2 LC + \sqrt{(RG - \omega^2 LC)^2 + \omega^2 (LG + RC)^2}}{2}} \qquad \ldots (1.39)$$

For a perfect transmission line $R = 0$ and $G = 0$,

$$\beta^2 = \omega^2 LC$$

$$\therefore \quad \beta = \omega\sqrt{LC} \qquad \text{[only positive value]}$$

Velocity :

The velocity of propagation is given by,

$$v = \lambda f$$

$$= 2\pi f \frac{\lambda}{2\pi}$$

$$v = \frac{\omega}{\beta} \qquad \left[\because \beta = \frac{2\pi}{\lambda} \text{ and } \omega = 2\pi f\right]$$

Substituting the value of $\beta = \omega\sqrt{LC}$

$$\therefore \quad v = \frac{\omega}{\omega\sqrt{LC}}$$

$$v = \frac{1}{\sqrt{LC}}$$

This is the velocity of propagation for an ideal line.

Wavelength :

The distance travelled by the wave along the line while the phase angle is changing through 2π radians is called wavelength.

$$\beta\lambda = 2\pi$$

$$\lambda = \frac{2\pi}{\beta} \quad \text{or} \quad \lambda = \frac{v}{f}$$

1.5. INPUT IMPEDANCE AND TRANSFER IMPEDANCE OF TRANSMISSION LINE

Input impedance :

The equations for voltage and current at the sending end of a transmission line of length 'l' are given by

$$V_S = V_R\left(\cosh\sqrt{ZY}\,l + \frac{Z_0}{Z_R}\sinh\sqrt{ZY}\,l\right) \quad \ldots (1.32)$$

$$I_S = I_R\left(\cosh\sqrt{ZY}\,l + \frac{Z_R}{Z_0}\sinh\sqrt{ZY}\,l\right) \quad \ldots (1.33)$$

The input impedance of the transmission line is,

$$Z_S = \frac{V_S}{I_S}$$

$$= \frac{V_R\left(\cosh\sqrt{ZY}\,l + \frac{Z_0}{Z_R}\sinh\sqrt{ZY}\,l\right)}{I_R\left(\cosh\sqrt{ZY}\,l + \frac{Z_R}{Z_0}\sinh\sqrt{ZY}\,l\right)}$$

$$= \frac{I_R Z_R\left(\cosh\sqrt{ZY}\,l + \frac{Z_0}{Z_R}\sinh\sqrt{ZY}\,l\right)}{I_R\left(\cosh\sqrt{ZY}\,l + \frac{Z_R}{Z_0}\sinh\sqrt{ZY}\,l\right)}$$

$$Z_S = \frac{Z_0\,(Z_R\cosh\sqrt{ZY}\,l + Z_0\sinh\sqrt{ZY}\,l)}{(Z_0\cosh\sqrt{ZY}\,l + Z_R\sinh\sqrt{ZY}\,l)} \quad \ldots (1.40)$$

Let $\sqrt{ZY} = \gamma$

The input impedance of the line is

$$Z_S = Z_0\left[\frac{Z_R\cosh\gamma l + Z_0\sinh\gamma l}{Z_0\cosh\gamma l + Z_R\sinh\gamma l}\right]$$

or $$Z_S = Z_0\left[\frac{Z_R + Z_0\tanh\gamma l}{Z_0 + Z_R\tanh\gamma l}\right].$$

In a different form, the equations for voltage and current at transmitting end of a line is given by equations (1.28) and (1.29),

$$V_S = \frac{V_R}{2}\left[\left(1 + \frac{Z_0}{Z_R}\right)e^{\sqrt{ZY}\,l} + \left(1 - \frac{Z_0}{Z_R}\right)e^{-\sqrt{ZY}\,l}\right] \quad \ldots (1.28)$$

$$I_S = \frac{I_R}{2}\left[\left(1 + \frac{Z_R}{Z_0}\right)e^{\sqrt{ZY}\,l} + \left(1 - \frac{Z_R}{Z_0}\right)e^{-\sqrt{ZY}\,l}\right] \quad \ldots (1.29)$$

or $$V_S = \frac{V_R}{2}\left[\left(\frac{Z_R+Z_0}{Z_R}\right)e^{\sqrt{ZY}\,l} + \left(\frac{Z_R-Z_0}{Z_R}\right)e^{-\sqrt{ZY}\,l}\right]$$

$$I_S = \frac{I_R}{2}\left[\left(\frac{Z_R+Z_0}{Z_0}\right)e^{\sqrt{ZY}\,l} + \left(\frac{Z_0-Z_R}{Z_0}\right)e^{-\sqrt{ZY}\,l}\right]$$

or $$V_S = \left(\frac{V_R}{2}\right)\left(\frac{Z_R+Z_0}{Z_R}\right)\left[e^{\sqrt{ZY}\,l} + \left(\frac{Z_R-Z_0}{Z_R+Z_0}\right)e^{-\sqrt{ZY}\,l}\right] \quad \ldots (1.41)$$

$$I_S = \frac{I_R}{2}\left(\frac{Z_R+Z_0}{Z_0}\right)\left[e^{\sqrt{ZY}\,l} - \left(\frac{Z_R-Z_0}{Z_R+Z_0}\right)e^{-\sqrt{ZY}\,l}\right] \quad \ldots (1.42)$$

The input impedance of the transmission line is given by,

$$Z_S = \frac{V_S}{I_S} = Z_0\left[\frac{e^{\sqrt{ZY}\,l} + \left(\frac{Z_R-Z_0}{Z_R+Z_0}\right)e^{-\sqrt{ZY}\,l}}{e^{\sqrt{ZY}\,l} - \left(\frac{Z_R-Z_0}{Z_R+Z_0}\right)e^{-\sqrt{ZY}\,l}}\right] \quad [\because V_R = I_R Z_R] \quad \ldots (1.43)$$

Let $\sqrt{ZY} = \gamma$

The input impedance of the transmission line is,

$$Z_S = Z_0\left[\frac{e^{\gamma l} + \left(\frac{Z_R-Z_0}{Z_R+Z_0}\right)e^{-\gamma l}}{e^{\gamma l} - \left(\frac{Z_R-Z_0}{Z_R+Z_0}\right)e^{-\gamma l}}\right] \quad \ldots (1.44)$$

If the line is terminated with its characteristic impedance *i.e.*, $Z_R = Z_0$, then the input impedance becomes equal to its characteristic impedance.

$$Z_S = Z_0$$

The input impedance of an infinite line is determined by letting $l \to \infty$.

$$\therefore \; Z_S = Z_0$$

It is found that a line of finite length, terminated with its characteristic impedance, appears to the transmitting end generator as an infinite line. A finite line terminated with Z_0 and an infinite line are same by measurements at the source.

If $K = \frac{Z_R - Z_0}{Z_R + Z_0}$, then

$$Z_S = Z_0\left[\frac{e^{\gamma l} + K e^{-\gamma l}}{e^{\gamma l} - K e^{-\gamma l}}\right] \quad \ldots (1.45)$$

Transfer impedance :

Transfer impedance is used to determine the current at the receiving end if voltage at transmitting end is known. Transfer impedance of a transmission line is defined as the ratio of voltage at the sending end (transmitted voltage) to the current at the receiving end (received current).

$$Z_T = \frac{V_S}{I_R}$$

Equation (1.41) becomes

$$V_S = \frac{V_R (Z_R + Z_0)}{2 Z_R} (e^{\gamma l} + K e^{-\gamma l})$$

$$V_S = \frac{I_R (Z_R + Z_0)}{2} (e^{\gamma l} + K e^{-\gamma l}) \qquad [\because V_R = I_R Z_R]$$

$$Z_T = \frac{V_S}{I_R} = \frac{Z_R + Z_0}{2} (e^{\gamma l} + K e^{-\gamma l})$$

$$= \frac{Z_R + Z_0}{2} \left(e^{\gamma l} + \frac{Z_R - Z_0}{Z_R + Z_0} e^{-\gamma l} \right)$$

$$= \left(\frac{Z_R + Z_0}{2} \right) e^{\gamma l} + \left(\frac{Z_R - Z_0}{2} \right) e^{-\gamma l}$$

$$= Z_R \left(\frac{e^{\gamma l} + e^{-\gamma l}}{2} \right) + Z_0 \left(\frac{e^{\gamma l} - e^{-\gamma l}}{2} \right)$$

$$= Z_R \cosh \gamma l + Z_0 \sinh \gamma l$$

$$\left[\because \frac{e^{\gamma l} + e^{-\gamma l}}{2} = \cosh \gamma l \quad \text{and} \quad \frac{e^{\gamma l} - e^{-\gamma l}}{2} = \sinh \gamma l \right]$$

$$Z_T = Z_R \cosh \gamma l + Z_0 \sinh \gamma l$$

1.6. LINE DISTORTION

Signal (*e.g.*, voice) transmitted over a transmission line is normally complex and consists of many frequency components. Such voice voltage will not have all frequencies transmitted with equal attenuation and equal time delay, the received waveform will not be identical with the input waveform at the sending end. This variation is known as ***distortion***. There are two types of line distortions. They are frequency distortion and delay distortion.

Frequency Distortion : A complex (voice) voltage transmitted on a transmission line will not be attenuated equally and the received waveform will not be identical with the input waveform at the transmitting end. This variation is known as frequency distortion.

The attenuation constant is given by

$$\text{Propagation constant } \gamma = \sqrt{(R + j\omega L)(G + j\omega C)}$$

$$= \sqrt{L\left(\frac{R}{L} + j\omega\right) C\left(\frac{G}{C} + j\omega\right)}$$

$$= \sqrt{LC}\sqrt{\left(\frac{R}{L} + j\omega\right)\left(\frac{G}{C} + j\omega\right)}$$

$$\text{But } \frac{R}{L} = \frac{G}{C}$$

$$\gamma = \sqrt{LC}\left(\frac{R}{L} + j\omega\right)$$

$$\text{Then} \quad \beta = \sqrt{\frac{\omega^2 LC - RG + RG + \omega^2 LC}{2}}$$

$$= \sqrt{\frac{2\omega^2 LC}{2}}$$

$$\beta = \omega\sqrt{LC}$$

Velocity of propagation is

$$v = \frac{\omega}{\beta}$$

$$v = \frac{1}{\sqrt{LC}}$$

This is the same velocity for all frequencies, thus eliminating delay distortion.

Attenuation factor

$$\alpha = \sqrt{\frac{RG - \omega^2 LC + \sqrt{(RG - \omega^2 LC)^2 + \omega^2 (LG + CR)^2}}{2}}$$

To make α is independent of frequency, the term $(RG - \omega^2 LC)^2 + \omega^2 (LG + CR)^2$ is forced to be equal to $(RG + \omega^2 LC)^2$.

$$(LG - CR)^2 = 0$$

$$LG = CR$$

$$\boxed{\frac{L}{C} = \frac{R}{G}}$$

This will make α and the velocity independent of frequency simultaneously. To achieve this condition, it requires a very large value of L, since G is small.

The attenuation factor

$$\alpha = \sqrt{\frac{RG - \omega^2 LC + \sqrt{(RG + \omega^2 LC)^2}}{2}}$$

$$= \sqrt{\frac{RG - \omega^2 LC + RG + \omega^2 LC}{2}}$$

$$= \sqrt{\frac{2RG}{2}}$$

$$\alpha = \sqrt{RG}$$

It is independent of frequency, thus eliminating frequency distortion on the line.

The characteristic impedance Z_0 is given by

$$Z_0 = \sqrt{\frac{R + j\omega L}{G + j\omega C}}$$

$$= \sqrt{\frac{L\left(\frac{R}{L} + j\omega\right)}{C\left(\frac{G}{C} + j\omega\right)}}$$

But $\frac{R}{L} = \frac{G}{C}$ for distortionless line.

$$\therefore \; Z_0 = \sqrt{\frac{L}{C}}$$

It is purely real and is independent of frequency.

1.8. TELEPHONE CABLE

In the telephone cable the wires are insulated with paper and twisted in pairs. This construction results in negligible values of inductance and conductance. Therefore $L\omega << R$ and $G << C\omega$.

$$Z = R + j\omega L \approx R$$

$$Y = G + j\omega C \approx j\omega C$$

Propagation constant $\gamma = \sqrt{ZY}$

$$= \sqrt{j\omega RC}$$

$$= \sqrt{\frac{j2\omega RC}{2}}$$

But $\gamma = \alpha + j\beta$

$$\alpha + j\beta = (1 + j)\sqrt{\frac{\omega RC}{2}}$$

Equating real and imaginary parts $\alpha = \sqrt{\frac{\omega RC}{2}}$

$$\beta = \sqrt{\frac{\omega RC}{2}}$$

Velocity of propagation $v = \frac{\omega}{\beta} = \frac{\omega}{\sqrt{\frac{\omega RC}{2}}} = \sqrt{\frac{2\omega}{RC}}$

The characteristic impedance $Z_0 = \sqrt{\frac{Z}{Y}} = \sqrt{\frac{R}{j\omega C}} = \sqrt{\frac{R}{\omega C}} \angle -45°$

It is found that the propagation constant α and velocity of propagation v are functions of frequency. Thus, the higher frequencies are attenuated more and travel faster than the lower frequencies resulting in considerable frequency and delay distortion.

1.9. LOADING OF LINES

It is necessary to increase L/C ratio to achieve distortionless condition in a transmission line. This can be done by increasing the inductance of a transmission line. Increasing inductance by inserting inductances in series with line is termed as ***loading*** and such lines are called ***loaded lines.*** The lumped inductors, known as ***loading coils*** are placed at suitable intervals along the transmission line to increase the effective distributed inductance.

The effect of loading can be realised by comparing the unloading of a transmission line in the attenuation Vs frequency graph. Fig.1.5 shows that the loaded line offers a low attenuation when compared to the unloaded line only for limited range of frequencies.

The important aspect of loading coil design is that saturation and stray fields should be avoided. It should have a low resistance and should be in small size. In general toroidal cores are used for loading coils.

Types of Loading

The open wire lines have more inductance of their own and so have much less distortion than cable. Therefore, the loading practice is not applicable to open wires but it is restricted to cables only. There are three types of loading in practice. They are

(a) Lumped loading

(b) Continuous loading

(c) Patch loading

(a) Lumped loading : The inductance of a transmission line can be increased by the introduction of loading coil at uniform intervals. This is called lumped loading. It acts as a low pass filter. So, it is applicable only for a limited range of frequency. The loading coils have an internal resistance R thus, increasing the total effective inductance increases R. Further hysteresis and eddy current losses which occur in the loading coils resulting in further apparent increase in R. Therefore, there is a practical limitation on the value of inductance that can be increased for the reduction of attenuation. Thus the loading coil should be carefully designed so that it will not introduce any distortion.

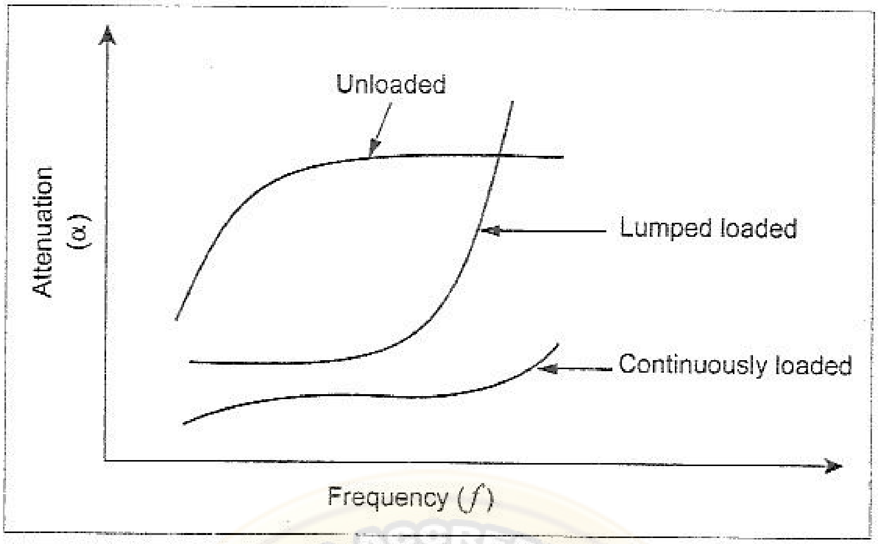


Fig. 1.5. Comparison of loaded and unloaded cable characteristics

(b) Continuous loading : A type of iron or some other magnetic material is wound on the transmission line (cable) to increase the permeability of the surrounding medium and thereby increase the inductance. It is a quite expensive method. Further eddy current and hysteresis losses in the magnetic material increases the primary constant R. Therefore, continuous loading is used only on ocean cables where lumped loading is difficult. The advantage of continuous loading over lumped loading is that attenuation factor α increases uniformly with increase in frequency.

(c) Patch loading : It employs sections of continuously loaded cable separated by sections of unloaded cable. The typical length for the section is normally a quarter kilometer. In this method the advantage of continuous loading is obtained and the cost is reduced considerably.

1.9.1. Inductance Loading of Telephone Cables

Distortionless line with distributed parameters is used to avoid the frequency and delay distortion experienced on telephone cables. It is necessary to increase the L/C to achieve distortionless condition $\frac{L}{C} = \frac{R}{G}$. Heaviside suggested that the inductance be increased and Pupin suggested that this increase in the inductance by lumped inductors spaced at intervals along the line. This use of inductance is called loading the line. The distributed loading is obtained by winding the cable with a high permeability steel tape such as permalloy in some submarine cables.

Consider an uniformly loaded cable with G = 0. Then,

$$Z = R + j\omega L$$

$$Y = j\omega C \qquad [\because G = 0]$$

$$Z = \sqrt{R^2 + (L\omega)^2} \;\underline{\Big|\, \tan^{-1}\left(\frac{L\omega}{R}\right)}$$

$$= \sqrt{R^2 + (L\omega)^2} \;\angle \frac{\pi}{2} - \tan^{-1}\frac{R}{L\omega}$$

Propagation constant $\gamma = \sqrt{ZY}$

$$= \sqrt{\sqrt{R^2 + (L\omega)^2} \;\angle \frac{\pi}{2} - \tan^{-1}\frac{R}{L\omega} \left(\omega C \angle \frac{\pi}{2}\right)}$$

$$= \sqrt{\omega C \sqrt{R^2 + (L\omega)^2} \;\angle \pi - \tan^{-1}\frac{R}{L\omega}}$$

$$= \sqrt{(\omega C)(L\omega)\sqrt{1 + \frac{R^2}{(L\omega)^2}}} \;\angle \frac{\pi}{2} - \frac{1}{2}\tan^{-1}\frac{R}{L\omega}$$

$$= \omega\sqrt{LC}\,\sqrt[4]{1 + \left(\frac{R}{L\omega}\right)^2} \;\angle \frac{\pi}{2} - \frac{1}{2}\tan^{-1}\frac{R}{L\omega}$$

Since R is small with respect to $L\omega$, the term $\left(\frac{R}{L\omega}\right)$ is neglected.

$$\therefore\; \gamma = \omega\sqrt{LC} \;\angle \frac{\pi}{2} - \frac{1}{2}\tan^{-1}\frac{R}{L\omega}$$

If $\theta = \frac{\pi}{2} - \frac{1}{2}\tan^{-1}\frac{R}{L\omega}$

$$\cos\theta = \cos\left(\frac{\pi}{2} - \frac{1}{2}\tan^{-1}\frac{R}{L\omega}\right)$$

$$= \sin\left(\frac{1}{2}\tan^{-1}\frac{R}{L\omega}\right)$$ $\frac{R}{2L\omega}$

For small angle,

$$\sin\theta \approx \tan\theta \approx \theta$$

so that $\cos\theta = \frac{R}{2L\omega}$

Similarly, $\sin\theta = \sin\left(\frac{\pi}{2} - \frac{1}{2}\tan^{-1}\frac{R}{L\omega}\right) = 1$

Propagation constant $\gamma = \omega\sqrt{LC}\;(\cos\theta + j\sin\theta)$

$$= \omega\sqrt{LC}\left(\frac{R}{2L\omega} + j\right)$$

$$\gamma = \frac{R\sqrt{LC}}{2L} + j\omega\sqrt{LC}$$

$$= \frac{R}{2}\sqrt{\frac{C}{L}} + j\omega\sqrt{LC}$$

$$\therefore \text{ Attenuation constant } \alpha = \frac{R}{2}\sqrt{\frac{C}{L}}$$

$$\text{Phase-shift } \beta = \omega\sqrt{LC}$$

$$\text{Velocity of propagation } v = \frac{\omega}{\beta} = \frac{1}{\sqrt{LC}}$$

It is noted that if G = 0 and Lω >> R, the attenuation and velocity are both independent of frequency and the loaded cable will be distortionless. Attenuation may be reduced by increasing L. Continuous (uniform) loading is expensive and achieves only a small increase in L per unit length. Lumped loading is preferred for cables.

Campbell's Equation

An analysis for the performance of a line loaded at uniform intervals can be obtained by considering a symmetrical section of line from the centre of one loading coil to the centre of the next coil. The section of line may be replaced with an equivalent T section having symmetrical series arms as shown in Fig.1.6. The series arm of T section including loading coil is given by

$$\frac{Z_1'}{2} = \frac{Z_c}{2} + \frac{Z_1}{2} \qquad \text{[From the fig.]}$$

where $\frac{Z_1}{2}$ is the series arm of T section.

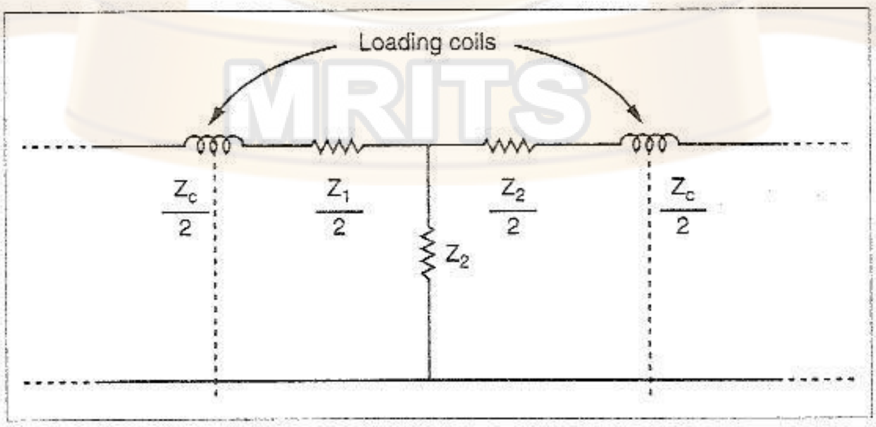


Fig. 1.6. Equivalent T section for part of a line between two lumped loading coils

$$\frac{Z_1}{2} = Z_0 \tanh\frac{\gamma l}{2}$$

$$\therefore \frac{Z_1'}{2} = \frac{Z_c}{2} + Z_0 \tanh\frac{\gamma l}{2}$$

where l is the distance between two loading coils.

The shunt Z_2 arm of the equivalent T section is

$$Z_2 = \frac{Z_0}{\sinh \gamma l}$$

For loaded T section

$$\cosh \gamma' l = 1 + \frac{Z_1'}{2 Z_2}$$

$$= 1 + \frac{\frac{Z_c}{2} + Z_0 \tanh \frac{\gamma l}{2}}{\frac{Z_0}{\sinh \gamma l}}$$

$$\text{But } \tanh \frac{\gamma l}{2} = \frac{\cosh \gamma l - 1}{\sinh \gamma l}$$

Substituting this value in above equation

$$\therefore \cosh \gamma' l = 1 + \frac{\frac{Z_c}{2} + Z_0 \frac{\cosh \gamma l - 1}{\sinh \gamma l}}{\frac{Z_0}{\sinh \gamma l}}$$

$$= 1 + \frac{\frac{Z_c}{2} \sinh \gamma l + Z_0 (\cosh \gamma l - 1)}{Z_0}$$

$$= 1 + \frac{Z_c}{2 Z_0} \sinh \gamma l + \cosh \gamma l - 1$$

$$\cosh \gamma' l = \frac{Z_c}{2 Z_0} \sinh \gamma l + \cosh \gamma l$$

This equation is called as Campbell's equation and it is used to determine the value of γ' of a line section consisting of partially lumped and partially distributed elements. For a cable Z_2 is capacitance and the cable capacitance and lumped inductances appear similar to the circuit of the low pass filter. It is found that for frequencies below cutoff, the attenuation is reduced, but the cut-off attenuation is increased (as a result of filter action). In practice, pure distortionless line is not obtained by loading, because R and L are to some extent functions of frequency. Eddy current losses are more in these coils. However, there is a major improvement in the loaded cable over the unloaded cable for a reasonable frequency range.

1.10. OPEN CIRCUITED AND SHORT CIRCUITED LINES

The expressions for voltage and current at the sending end of a transmission line of length 'l' are given by

$$V_S = V_R\left[\cosh\sqrt{ZY}\,l + \frac{Z_o}{Z_R}\sinh\sqrt{ZY}\,l\right]$$

$$I_S = I_R\left[\cosh\sqrt{ZY}\,l + \frac{Z_R}{Z_o}\sinh\sqrt{ZY}\,l\right]$$

The input impedance of a transmission line is given by

$$Z_S = \frac{V_S}{I_S}$$

$$= \frac{V_R\left[\cosh\sqrt{ZY}\,l + \frac{Z_o}{Z_R}\sinh\sqrt{ZY}\,l\right]}{I_R\left[\cosh\sqrt{ZY}\,l + \frac{Z_R}{Z_o}\sinh\sqrt{ZY}\,l\right]}$$

$$= \frac{V_R}{I_R}\,\frac{Z_o}{Z_R}\,\frac{(Z_R\cosh\gamma l + Z_o\sinh\gamma l)}{(Z_o\cosh\gamma l + Z_R\sinh\gamma l)}$$

$$= Z_o\left(\frac{Z_R\cosh\gamma l + Z_o\sinh\gamma l}{Z_o\cosh\gamma l + Z_R\sinh\gamma l}\right) \qquad \left[\because Z_R = \frac{V_R}{I_R}\right]$$

$$Z_S = Z_o\left(\frac{Z_R\cosh\gamma l + Z_o\sinh\gamma l}{Z_o\cosh\gamma l + Z_R\sinh\gamma l}\right)$$

If short circuited, the receiving end impedance is zero.

$$i.e.,\quad Z_R = 0$$

$$\therefore Z_{sc} = Z_o\left(\frac{Z_o\sinh\gamma l}{Z_o\cosh\gamma l}\right)$$

Short circuited impedance

$$Z_{sc} = Z_o\tanh\gamma l$$

If open circuited, the receiving end impedance is infinite.

i.e., $Z_R = \infty$

Input impedance of transmission line can be written as

$$Z_S = Z_o\left[\frac{\cosh\gamma l + \frac{Z_o}{Z_R}\sinh\gamma l}{\frac{Z_o}{Z_R}\cosh\gamma l + \sinh\gamma l}\right]$$

Applying $Z_R = \infty$

$$\text{Then} \quad Z_{oc} = Z_0 \left[\frac{\cosh \gamma l}{\sinh \gamma l} \right]$$

The open circuited impedance

$$Z_{oc} = Z_0 \coth \gamma l$$

By multiplying open circuited impedance and short circuited impedances

$$Z_{oc} Z_{sc} = Z_0^2 \tanh \gamma l \coth \gamma l$$

$$= Z_0^2$$

The characteristic impedance is given by

$$Z_0 = \sqrt{Z_{oc} Z_{sc}}$$

By dividing short circuited impedance by open circuited impedance.

$$\frac{Z_{sc}}{Z_{oc}} = \frac{Z_0 \tanh \gamma l}{Z_0 \coth \gamma l}$$

$$= \tan^2 h \, \gamma l$$

$$\tanh \gamma l = \sqrt{\frac{Z_{sc}}{Z_{oc}}}$$

$$\gamma l = \tanh^{-1} \sqrt{\frac{Z_{sc}}{Z_{oc}}}$$

1.11. REFLECTION

When the load impedance is not equal to the characteristic impedance of transmission line, reflection takes place.

The expressions for voltage and current on the transmission line are

$$V = \frac{V_R}{2} \left[\left(1 + \frac{Z_0}{Z_R}\right) e^{\sqrt{ZY}\,x} + \left(1 - \frac{Z_0}{Z_R}\right) e^{-\sqrt{ZY}\,x} \right]$$

$$I = \frac{I_R}{2} \left[\left(1 + \frac{Z_R}{Z_0}\right) e^{\sqrt{ZY}\,x} + \left(1 - \frac{Z_R}{Z_0}\right) e^{-\sqrt{ZY}\,x} \right]$$

or

$$V = \frac{V_R}{2} \left[\frac{Z_R + Z_0}{Z_R} e^{\sqrt{ZY}\,x} + \frac{Z_R - Z_0}{Z_R} e^{-\sqrt{ZY}\,x} \right]$$

$$I = \frac{I_R}{2} \left[\frac{Z_R + Z_0}{Z_0} e^{\sqrt{ZY}\,x} - \frac{Z_R - Z_0}{Z_0} e^{-\sqrt{ZY}\,x} \right]$$

or

$$V = \frac{V_R (Z_R + Z_o)}{2 Z_R} \left[e^{\gamma x} + \left(\frac{Z_R - Z_o}{Z_R + Z_o} \right) e^{-\gamma x} \right]$$

$$I = \frac{I_R (Z_R + Z_o)}{2 Z_R} \left[e^{\gamma x} - \left(\frac{Z_R - Z_o}{Z_R + Z_o} \right) e^{-\gamma x} \right]$$

$$[\because \gamma = \sqrt{ZY}]$$

If the transmission line is not terminated with the characteristic impedance *i.e.*, $Z_R \neq Z_o$ (mismatch) the above expressions for voltage and current exist. It consists of two waves, one is moving in the forward (positive x) direction which is called incident wave and the other is moving in the opposite (negative x) direction which is called reflected ray. The term varying with $e^{\gamma x}$ represents a wave progressing from the sending end towards the receiving end and the amplitude decreasing with increased distance. The term varying with $e^{-\gamma x}$ represents a wave progressing from the receiving end towards the sending end, decreasing in amplitude with increased distance.

If the transmission line is terminated with characteristic impedance *i.e.*, $Z_R = Z_o$ (properly matched) then the voltage and current expressions are

$$V = V_R e^{\gamma x}$$

$$I = I_R e^{\gamma x}$$

The incident wave moves only in forward (positive x) direction. There is no reflected wave in the opposite direction.

1.11.1. Reflection Coefficient

Reflection coefficient is defined as the ratio of the reflected voltage to the incident voltage at the receiving end of the line.

$$K = \frac{\text{Reflected voltage at load}}{\text{Incident voltage at load}} = \frac{V_R}{V_S}$$

The equation for the voltage of a transmission line is

$$V = \frac{V_R (Z_R + Z_o)}{2 Z_R} \left[e^{\gamma x} + \left(\frac{Z_R - Z_o}{Z_R + Z_o} \right) e^{-\gamma x} \right]$$

$$V = \frac{V_R (Z_R + Z_o)}{2 Z_R} e^{\gamma x} + \frac{V_R (Z_R - Z_o)}{2 Z_R} e^{-\gamma x}$$

The first term ($e^{\gamma x}$) represents incident wave, whereas the second term ($e^{-\gamma x}$) represents the reflected wave. The ratio of amplitude of the reflected wave voltage to the amplitude of the incident wave voltage is nothing but reflection coefficient.

$$K = \frac{\frac{V_R (Z_R - Z_o)}{2 Z_R}}{\frac{V_R (Z_R + Z_o)}{2 Z_R}} = \frac{Z_R - Z_o}{Z_R + Z_o}$$

$$\boxed{K = \frac{Z_R - Z_o}{Z_R + Z_o}}$$

It is also defined as in terms of the ratio of the reflected current to the incident current. But it is negative.

$$-K = \frac{\text{Reflected current at load}}{\text{Incident current at load}} = \frac{I_R}{I_S}$$

If the transmission line is terminated by its characteristic impedance ($Z_R = Z_o$), the reflection coefficient becomes zero.

1.11.2. Reflection Factor and Reflection Loss

Consider a transmission line with a voltage source V_S and its impedance Z_1 and load impedance Z_2 as shown in Fig.1.7. If Z_2 is not equal to Z_1, reflection takes place. The power delivered to the load is less than that with impedance matching. Reflection results in power loss. This loss is known as reflection loss.

Fig. 1.7. Transmission line with voltage source V_S and impedance Z_1

Image matching between the impedances Z_1 and Z_2 can be obtained by inserting an ideal transformer and a phase shifting network between Z_1 and Z_2. If I_1 and I_2 be the currents in the primary and secondary of the transformer respectively, the current ratio of the transformer is given by

$$\frac{I_2}{I_1} = \sqrt{\frac{Z_1}{Z_2}}$$

Z_2 may be adjusted to that of Z_1 by choosing the proper transformation ratio and phase angle. Z_2 is the image impedance of Z_1. The current through the source is

$$I_1 = \frac{V_S}{2 Z_1}$$

The current flow in the secondary of the transformer under image impedance matching is

$$I_2' = I_1 \sqrt{\frac{Z_1}{Z_2}} = \frac{V}{2 Z_1} \sqrt{\frac{Z_1}{Z_2}} = \frac{V_S}{2\sqrt{Z_1 Z_2}}$$

The current in the load impedance Z_2 without image matching.

$$|I_2| = \frac{|V_S|}{|Z_1 + Z_2|}$$

The ratio of the current actually flowing in the load to that which might flow under matched condition is known as ***reflection factor.***

$$\left|\frac{I_2}{I_2'}\right| = \frac{\dfrac{|V_S|}{|Z_1 + Z_2|}}{\dfrac{|V_S|}{|2\sqrt{Z_1 Z_2}|}}$$

$$k = \left|\frac{2\sqrt{Z_1 Z_2}}{Z_1 + Z_2}\right|$$

The reflection factor indicates the change in current in the load due to reflection at the mismatched junction.

The ***reflection loss*** is the reciprocal of the reflection factor in nepers or dB.

$$\text{Reflection loss} = ln\frac{1}{k}$$

$$= ln\left|\frac{Z_1 + Z_2}{2\sqrt{Z_1 Z_2}}\right| \text{ nepers}$$

$$= 20 \log\left|\frac{Z_1 + Z_2}{2\sqrt{Z_1 Z_2}}\right| \text{ dB}$$

1.12. T AND π SECTIONS EQUIVALENT TO LINES

A T section is shown in Fig.1.8 with two ports 1, 1 and 2, 2.

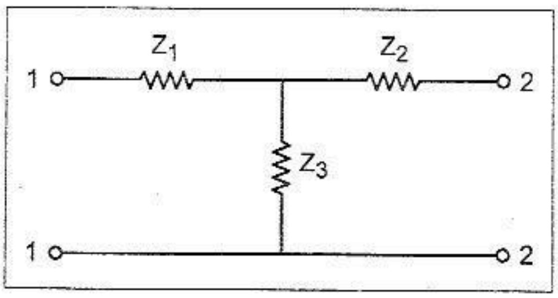


Fig. 1.8. T section network

Impedance measurements may be made at any port with the other port opened or shorted.

Let Z_{1OC} be the impedance at port 1 when port 2 is open circuited.

Z_{1SC} be the impedance at port 1 when port 2 is short circuited.

Z_{2OC} be the impedance at port 2 when port 1 is open circuited.

Z_{2SC} be the impedance at port 2 when port 1 is short circuited.

$$Z_{1OC} = Z_1 + Z_3$$

$$Z_{1SC} = Z_1 + \frac{Z_2 Z_3}{Z_2 + Z_3}$$

$$Z_{2OC} = Z_2 + Z_3$$

$$Z_{2SC} = Z_2 + \frac{Z_1 Z_3}{Z_1 + Z_3}$$

By solving these equations, the values of Z_1, Z_2 and Z_3 are determined.

$$\begin{aligned}
Z_{1OC} - Z_{1SC} &= Z_3 - \frac{Z_2 Z_3}{Z_2 + Z_3} \\
&= \frac{Z_3 Z_2 + Z_3^2 - Z_2 Z_3}{Z_2 + Z_3} \\
&= \frac{Z_3^2}{Z_2 + Z_3} \\
&= \frac{Z_3^2}{Z_{2OC}} \qquad [\because Z_2 + Z_3 = Z_{2OC}] \\
Z_3^2 &= Z_{2OC}(Z_{1OC} - Z_{1SC}) \\
Z_3 &= \pm\sqrt{Z_{2OC}(Z_{1OC} - Z_{1SC})}
\end{aligned}$$

Taking the positive value,

$$\begin{aligned}
Z_3 &= \sqrt{Z_{2OC}(Z_{1OC} - Z_{1SC})} \\
Z_1 &= Z_{1OC} - Z_3 \qquad [\because Z_{1OC} = Z_1 + Z_3] \\
&= Z_{1OC} - \sqrt{Z_{2OC}(Z_{1OC} - Z_{1SC})} \\
Z_2 &= Z_{2OC} - Z_3 \qquad [\because Z_{2OC} = Z_2 + Z_3] \\
&= Z_{2OC} - \sqrt{Z_{2OC}(Z_{1OC} - Z_{1SC})} \\
Z_1 &= Z_{1OC} - \sqrt{Z_{2OC}(Z_{1OC} - Z_{1SC})} \\
Z_2 &= Z_{2OC} - \sqrt{Z_{2OC}(Z_{1OC} - Z_{1SC})}
\end{aligned}$$

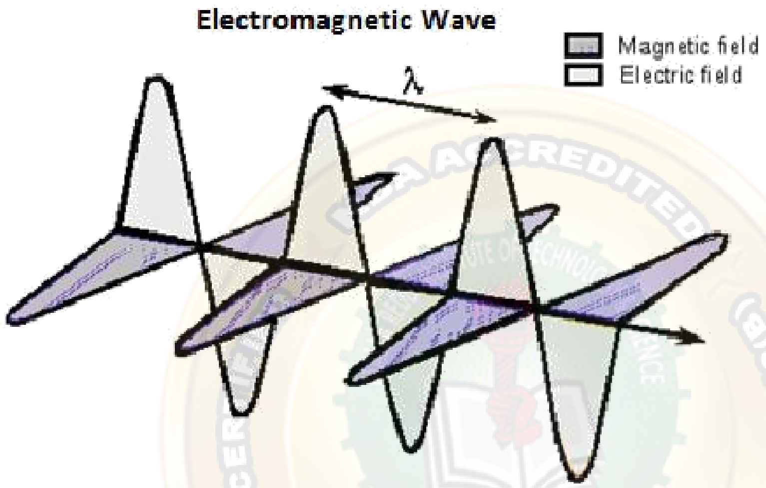
Transmission Lines – II

- SC and OC Lines
- Input Impedance Relations
- Reflection Coefficient
- VSWR
- $\lambda/4$, $\lambda/2$, $\lambda/8$ Lines - Impedance Transformations
- Smith Chart - Configuration and Applications,
- Single Stub Matching
- Illustrative Problems.

This means, more the current flows towards the surface of the conductor, it flows less towards the center, which is known as the **Skin Effect**.

Inductance

In an AC transmission line, the current flows sinusoidally. This current induces a magnetic field perpendicular to the electric field, which also varies sinusoidally. This is well known as Faraday's law. The fields are depicted in the following figure.



This varying magnetic field induces some EMF into the conductor. Now this induced voltage or EMF flows in the opposite direction to the current flowing initially. This EMF flowing in the opposite direction is equivalently shown by a parameter known as **Inductance**, which is the property to oppose the shift in the current.

It is denoted by "**L**". The unit of measurement is "**Henry** H".

Conductance

There will be a leakage current between the transmission line and the ground, and also between the phase conductors. This small amount of leakage current generally flows through the surface of the insulator. Inverse of this leakage current is termed as **Conductance**. It is denoted by "**G**".

The flow of line current is associated with inductance and the voltage difference between the two points is associated with capacitance. Inductance is associated with the magnetic field, while capacitance is associated with the electric field.

Capacitance

The voltage difference between the **Phase conductors** gives rise to an electric field between the conductors. The two conductors are just like parallel plates and the air in between them becomes dielectric. This pattern gives rise to the capacitance effect between the conductors.

Characteristic Impedance

If a uniform lossless transmission line is considered, for a wave travelling in one direction, the ratio of the amplitudes of voltage and current along that line, which has no reflections, is called as **Characteristic impedance**.

It is denoted by Z_0

$$Z_0 = \sqrt{\frac{voltage\ \ wave\ \ value}{current\ \ wave\ \ value}}$$

$$Z_0 = \sqrt{\frac{R + jwL}{G + jwC}}$$

For a lossless line, $R_0 = \sqrt{\frac{L}{C}}$

Where L & C are the inductance and capacitance per unit lengths.

Impedance Matching

To achieve maximum power transfer to the load, impedance matching has to be done. To achieve this impedance matching, the following conditions are to be met.

The resistance of the load should be equal to that of the source.

$$R_L = R_S$$

The reactance of the load should be equal to that of the source but opposite in sign.

$$X_L = -X_S$$

Which means, if the source is inductive, the load should be capacitive and vice versa.

Reflection Co-efficient

The parameter that expresses the amount of reflected energy due to impedance mismatch in a transmission line is called as **Reflection coefficient**. It is indicated by $\rho\, rho$.

It can be defined as "the ratio of reflected voltage to the incident voltage at the load terminals".

$$\rho = \frac{reflected\ voltage}{incident\ voltage} = \frac{V_r}{V_i}\ at\ load\ terminals$$

If the impedance between the device and the transmission line don't match with each other, then the energy gets reflected. The higher the energy gets reflected, the greater will be the value of ρ reflection coefficient.

Voltage Standing Wave Ratio $VSWR$

The standing wave is formed when the incident wave gets reflected. The standing wave which is formed, contains some voltage. The magnitude of standing waves can be measured in terms of standing wave ratios.

The ratio of maximum voltage to the minimum voltage in a standing wave can be defined as Voltage Standing Wave Ratio $VSWR$. It is denoted by "S".

$$S = \frac{|V_{max}|}{|V_{min}|} \quad 1 \leq S \leq \infty$$

VSWR describes the voltage standing wave pattern that is present in the transmission line due to phase addition and subtraction of the incident and reflected waves.

Hence, it can also be written as

$$S = \frac{1+\rho}{1-\rho}$$

The larger the impedance mismatch, the higher will be the amplitude of the standing wave. Therefore, if the impedance is matched perfectly,

$$V_{max} : V_{min} = 1:1$$

Hence, the value for VSWR is unity, which means the transmission is perfect.

Efficiency of Transmission Lines

The efficiency of transmission lines is defined as the ratio of the output power to the input power.

$$\%\ efficiency\ of\ transmission\ line\ \eta = \frac{Power\ delivered\ at\ reception}{Power\ sent\ from\ the\ transmission\ end} \times 100$$

Voltage Regulation

Voltage regulation is defined as the change in the magnitude of the voltage between the sending and receiving ends of the transmission line.

$$\%\ voltage\ regulation = \frac{sending\ end\ voltage - receiving\ end\ voltage}{sending\ end\ voltage} \times 100$$

Losses due to Impedance Mismatch

The transmission line, if not terminated with a matched load, occurs in losses. These losses are many types such as attenuation loss, reflection loss, transmission loss, return loss, insertion loss, etc.

Attenuation Loss

The loss that occurs due to the absorption of the signal in the transmission line is termed as Attenuation loss, which is represented as

$$Attenuation\ loss(dB) = 10\ log_{10}\left[\frac{E_i - E_r}{E_t}\right]$$

Where

- E_i = the input energy
- E_r = the reflected energy from the load to the input
- E_t = the transmitted energy to the load

Reflection Loss

The loss that occurs due to the reflection of the signal due to impedance mismatch of the transmission line is termed as Reflection loss, which is represented as

$$Reflection\ loss(dB) = 10\ log_{10}\left[\frac{E_i}{E_i - E_r}\right]$$

Where

- E_i = the input energy
- E_r = the reflected energy from the load

Transmission Loss

The loss that occurs while transmission through the transmission line is termed as Transmission loss, which is represented as

$$Transmission\ loss(dB) = 10\ log_{10}\ \frac{E_i}{E_t}$$

Where

- E_i = the input energy
- E_t = the transmitted energy

Return Loss

The measure of the power reflected by the transmission line is termed as Return loss, which is represented as

$$Return\ loss(dB) = 10\ log_{10}\ \frac{E_i}{E_r}$$

Where

- E_i = the input energy
- E_r = the reflected energy

Insertion Loss

The loss that occurs due to the energy transfer using a transmission line compared to energy transfer without a transmission line is termed as Insertion loss, which is represented as

$$Insertion\ loss(dB) = 10\ log_{10}\ \frac{E_1}{E_2}$$

Where

- E_1 = the energy received by the load when directly connected to the source, without a transmission line.
- E_2 = the energy received by the load when the transmission line is connected between the load and the source.

Stub Matching

If the load impedance mismatches the source impedance, a method called "Stub Matching" is sometimes used to achieve matching.

The process of connecting the sections of open or short circuit lines called **stubs** in the shunt with the main line at some point or points, can be termed as **Stub Matching**.

At higher microwave frequencies, basically two stub matching techniques are employed.

Single Stub Matching

In Single stub matching, a stub of certain fixed length is placed at some distance from the load. It is used only for a fixed frequency, because for any change in frequency, the location of the stub has to be changed, which is not done. This method is not suitable for coaxial lines.

Double Stub Matching

In double stud matching, two stubs of variable length are fixed at certain positions. As the load changes, only the lengths of the stubs are adjusted to achieve matching. This is widely used in laboratory practice as a single frequency matching device.

The following figures show how the stub matchings look.

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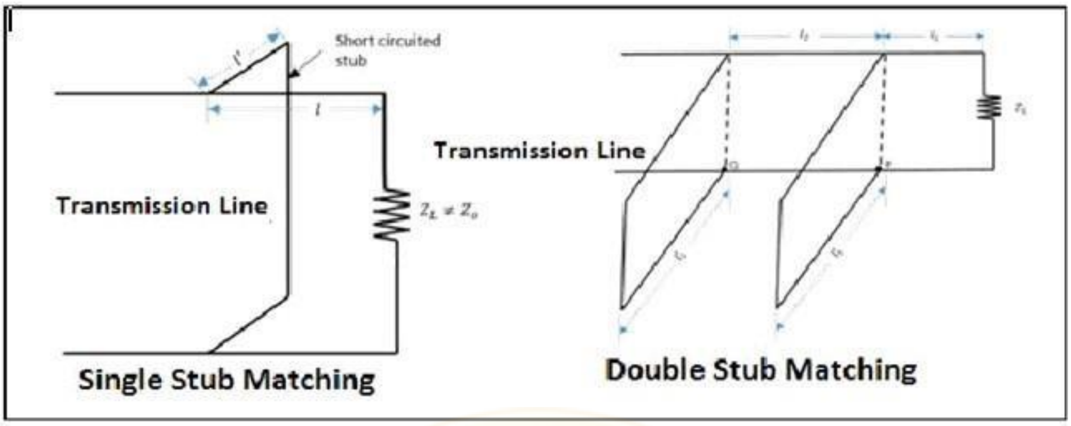

Short circuited stub
Transmission Line
$Z_L \neq Z_o$
Single Stub Matching
Transmission Line
Double Stub Matching

Transmission Lines – Smith Chart & Impedance Matching (Intensive Reading)

1 The Smith Chart

Transmission line calculations – such as the determination of input impedance using equation (4.30) and the reflection coefficient or load impedance from equation (4.32) – often involves tedious manipulation of complex numbers. This tedium can be alleviated using a graphical method of solution. The best known and most widely used graphical chart is the Smith chart. The Smith chart is a circular plot with a lot of interlaced circles on it. When used correctly, impedance matching can be performed without any computation. The only effort required is the reading and following of values along the circles.

The Smith chart is a polar plot of the complex reflection coefficient, or equivalently, a graphical plot of normalized resistance and reactance functions in the reflection-coefficient plane. To understand how the Smith chart for a lossless transmission line is constructed, examine the voltage reflection coefficient of the load impedance defined by

$$\Gamma_L = \frac{V_{\text{refl}}}{V_{\text{inc}}} = \frac{Z_L - Z_0}{Z_L + Z_0} = \Gamma_{re} + j\Gamma_{im}, \tag{1}$$

where Γ_{re} and Γ_{im} are the real and imaginary parts of the complex reflection coefficient Γ_L. The characteristic impedance Z_0 is often a constant and a real industry normalized value, such as 50 ∧, 75 ∧, 100 ∧, and 600 ∧. We can then define the normalised load impedance by

$$z_L = Z_L / Z_0 = (R + jX) / Z_0 = r + jx. \tag{2}$$

With this simplification, we can rewrite the reflection coefficient formula in (1) as

$$\Gamma_L = \Gamma_{re} + j\Gamma_{im} = \frac{(Z_L - Z_0)/Z_0}{(Z_L + Z_0)/Z_0} = \frac{z_L - 1}{z_L + 1}. \tag{3}$$

The inverse relation of (3) is

$$z_L = \frac{1+\Gamma_L}{1-\Gamma_L} = \frac{1+|\Gamma_L| e^{j\theta}}{1-|\Gamma_L| e^{j\theta}} \tag{4}$$

or

$$r + jx = \frac{(1+\Gamma_{re}) + j\Gamma_{im}}{(1-\Gamma_{re}) - j\Gamma_{im}}. \tag{5}$$

Multiplying both the numerator and the denominator of (5) by the complex conjugate of the denominator and separating the real and imaginary parts, we obtain

$$r = \frac{1 - \Gamma_{re}^2 - \Gamma_{im}^2}{(1 - \Gamma_{re})^2 + \Gamma_{im}^2} \quad (6)$$

and

$$x = \frac{2\Gamma_{im}^2}{(1 - \Gamma_{re})^2 + \Gamma_{im}^2}. \quad (7)$$

Equation (6) can be rearranged as

$$\left(\Gamma_{re} - \frac{r}{1+r}\right)^2 + \Gamma_{im}^2 = \left(\frac{1}{1+r}\right)^2. \quad (8)$$

This equation is a relationship in the form of a parametric equation $(x-a)^2 + (y-b)^2 = R^2$ in the complex plane $(\Gamma_{re}, \Gamma_{im})$ of a circle centred at the coordinates $\left(\frac{r}{r+1}, 0\right)$ and having a radius of $\frac{1}{r+1}$. Different values of r yield circles of different radii with centres at different positions on the Γ_{re}-axis. The following properties of the r-circles are noted:

- The centres of all r-circles lie on the Γ_{re}-axis.
- The circle where there is no resistance ($r = 0$) is the largest. It is centred at the origin andhas a radius of 1.
- The r-circles become progressively smaller as r increases from 0 to ∞, ending at the$(\Gamma_{re} = 1, \Gamma_{im} = 0)$ point for an open circuit.
- All the r-circles pass through the point $(\Gamma_{re} = 1, \Gamma_{im} = 0)$.

See Figure 1 for further details.

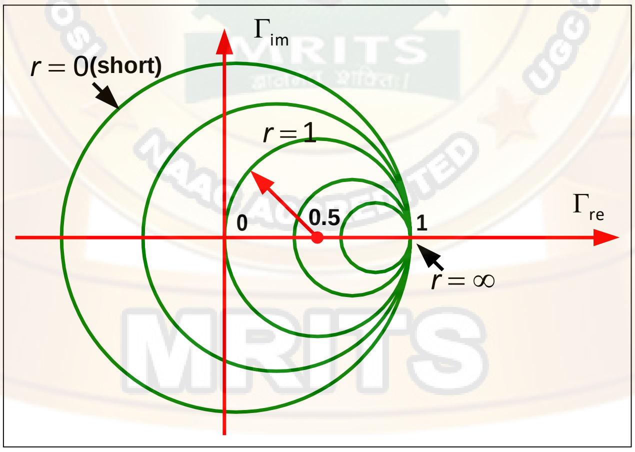


Figure 1: The r-circles in the complex plane $(\Gamma_{re}, \Gamma_{im})$.

Similarly, (7) can be rearranged as

$$(\Gamma_{re} - 1)^2 + \left(\Gamma_{im} - \frac{1}{x}\right)^2 = \left(\frac{1}{x}\right)^2 . \qquad (9)$$

Again, (9) is a parametric equation of the type $(x-a)^2 + (y-b)^2 = R^2$ in the complex plane

(Γ_r, Γ_i) of a circle centred at the coordinates $\left(1, \frac{1}{x}\right)$ and having a radius of $\frac{1}{x}$. Different values of x yield circles of different radii with centres at different positions on the $\Gamma_{re} = 1$ line. The following properties of the x-circles are noted:

- The centres of all x-circles lie on the $\Gamma_{re} = 1$ line; those for $x > 0$ (inductive reactance) lieabove the Γ_{re} -axis, and those for $x < 0$ lie below the Γ_{re} -axis.
- The $x = 0$ circle becomes the Γ_{re}-axis.
- The x-circles become progressively smaller as $|x|$ increases from 0 to ∞, ending at the ($\Gamma_{re} = 1$, $\Gamma_{im} = 0$) point for an open circuit.
- All the x-circles pass through the point $(\Gamma_{re} = 1, \Gamma_{im} = 0)$.

See Figure 2 for further details.

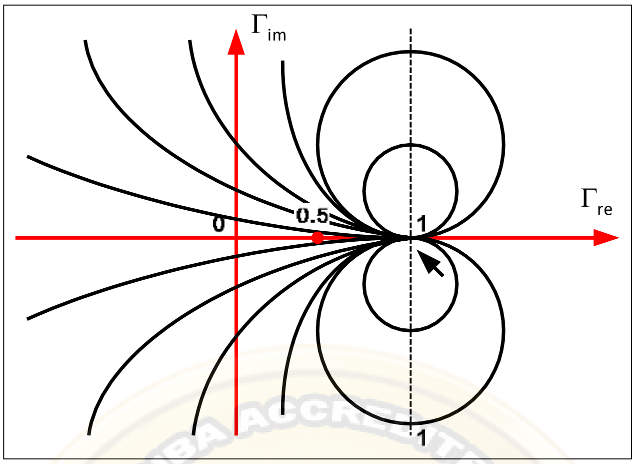


Figure 2: The x-circles in the complex plane $(\Gamma_{re}, \Gamma_{im})$.

To complete the Smith chart, the two circles' families are superimposed. The Smith chart therefore becomes a chart of r- and x-circles in the $(\Gamma_{re}, \Gamma_{im})$ -plane for $|\Gamma| \leq 1$. The intersection of an r-circle and an x-circle defines a point which represents a normalized load impedance $z_L = r + j\,x$. The actual load impedance is $Z_L = Z_0 z_L = Z_0 (r + j\,x)$. As an illustration, the impedance $Z_L = 85 + j30$ in a $Z_0 = 50\ \wedge$ -system is represented by the point P in Figure 3. Here $z_L = 1.7 + j\,0.6$ at the intersection of the $r = 1.7$ and the $x = 0.6$ circles. Values for Γ_{re} and Γ_{im} may then be obtained from the projections onto the horizontal and vertical axes (see Figure 4). These are approximately given by $\Gamma_{re} \approx 0.3$ and $\Gamma_{im} \approx 0.16$.

Point P_{sc} at $(\Gamma_{re} = -1, \Gamma_{im} = 0)$ corresponds to $r = 0$ and $x = 0$ and therefore represents a short-circuit. P_{oc} at $(\Gamma_{re} = 1, \Gamma_{im} = 0)$ corresponds to an infinite impedance therefore represents an open circuit.

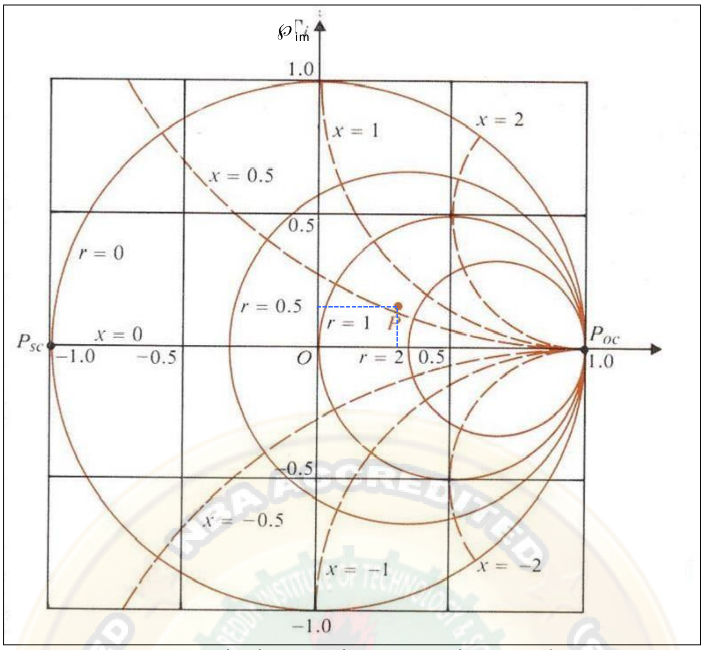


Figure 3: Smith chart with rectangular coordinates.

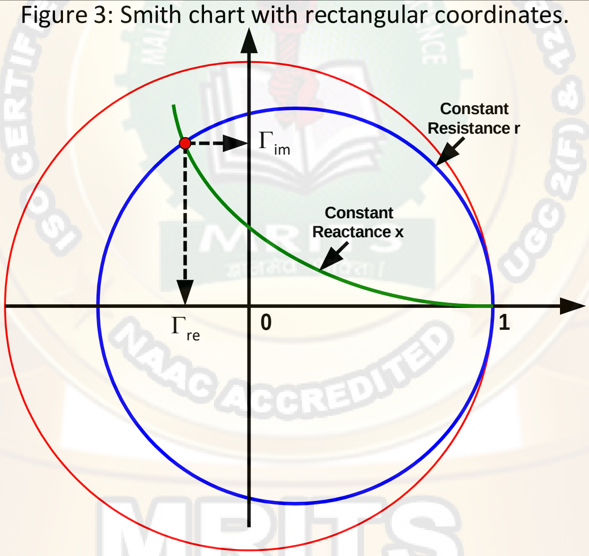


Figure 4: Direct extraction of the reflection coefficient $\Gamma = \Gamma_{re} + j\Gamma_{im}$ along the horizontal and vertical axes.

Instead of having a Smith chart marked with Γ_{re} and Γ_{im} marked in rectangular coordinates, the same chart can be marked in polar coordinates, so that every point in the Γ-plane is specified by a magnitude $|\Gamma|$ and a phase angle θ. This is illustrated in Figure 5, where several $|\Gamma|$-circles are shown in dashed lines and some θ-angles are marked around the $|\Gamma| = 1$ circle. The $|\Gamma|$-circles are normally not shown on commercially available Smith charts, but once the point representing a certain $z_L = r + jx$ is located, it is simply a matter of drawing a circle centred at the origin through the point. The ratio of the distance to the point and the radius to the edge of the chart is equal to the magnitude of $|\Gamma|$ of the load reflection coefficient, and the angle that a line to that point makes with the real axis represents θ . If, for example the point

$Z_1 = 100 + j50 \wedge$	$Z_2 = 75 - j100 \wedge$	$Z_3 = j200 \wedge$	$Z_4 = 150 \wedge$
$Z_5 = \infty$ (an open circuit)	$Z_6 = 0$ (a short circuit)	$Z_7 = 50 \wedge$	$Z_8 = 184 - j900 \wedge$

The normalized impedances shown below are plotted in Figure 6.

$z_1 = 2 + j$	$z_2 = 1.5 - j2$	$z_3 = j4$	$z_4 = 3$
$z_5 = \infty$	$z_6 = 0$	$z_7 = 1$	$z_8 = 3.68 - j18$

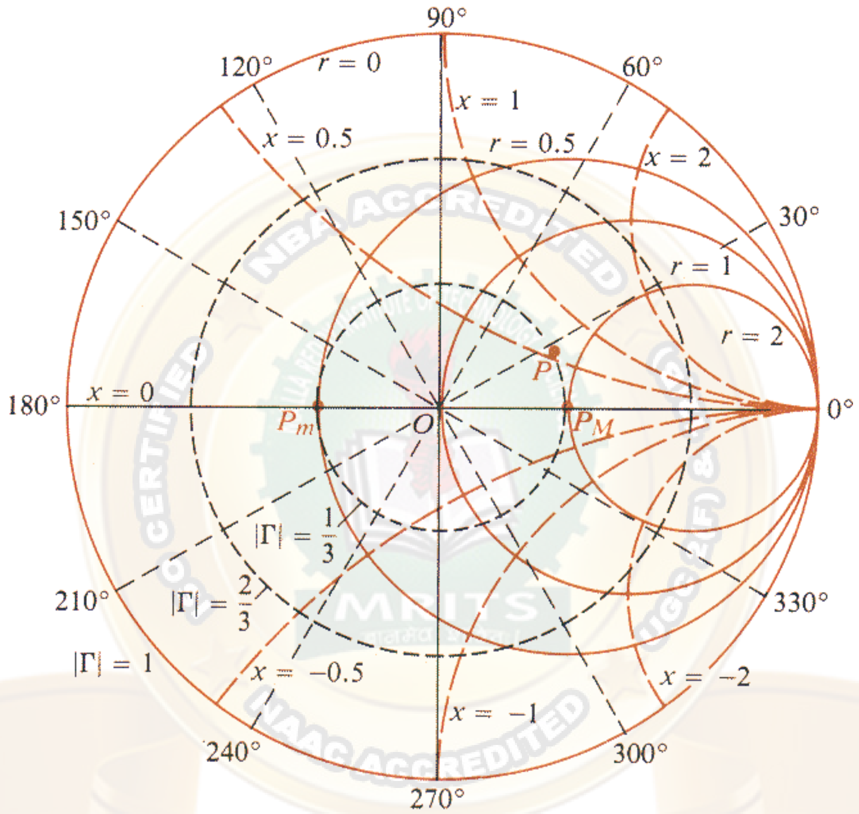
It is also possible to directly extract the reflection coefficient Γ on the Smith chart of Figure 6. Once the impedance point is plotted (the intersection point of a constant resistance circle and

$z_L = 1.7 + j0.6$ is marked on the Smith chart at point *P*, we find that

$|\Gamma_L| = 1/3$ and $\theta = 28°$.

Each $|\Gamma|$ circle intersects the real axis at two points. In Figure 5 we designate the point on the

positive real axis as P_M and on the negative real axis as P_m. Since $x = 0$ along the real axis, both these points represent situations of a purely resistive load, $Z_L = R_L$. Obviously, $R_L > Z_0$ at P_M where $r > 1$, and $R_L < Z_0$ at P_m where $r < 1$. Since $S = R_L / Z_0$ for $R_L > Z_0$, the value of the r-circle passing through the point P_M is numerically equal to the standing wave ratio. For the



example where $z_L = 1.7 + j0.6$, we find that $r = 2$ at P_M , so that $S = r = 2$.

Figure 5: Smith chart in polar coordinates.

Example 1:

Consider a characteristic impedance of 50 $\wedge$ with the following impedances:

of a constant reactance circle), simply read the rectangular coordinates projection on the horizontal and vertical axis. This will give Γ_{re} , the real part of the reflection coefficient, and

Γ_{im} , the imaginary part of the reflection coefficient. Alternatively, the reflection coefficient may be obtained in polar form by using the scales provided on the commercial Smith chart.

$\Gamma_1 = 0.4 + 0.2\,j$ $= 0.45 \angle 27°$ $\quad$ $\Gamma_2 = 0.51 - 0.4\,j$ $= 0.65 \angle -38°$ $\quad$ $\Gamma_3 = 0.875 + 0.48j$ $= 0.998 \angle 29°$ $\quad$ $\Gamma_4 = 0.5$ $= 0.5 \angle 0°$

$\Gamma_5 = 1$ $= 1 \angle 0°$ $\quad$ $\Gamma_6 = -1$ $= 1 \angle 180°$ $\quad$ $\Gamma_7 = 0$ $= 0$ $\quad$ $\Gamma_8 = 0.96 - 0.1\,j$ $= 0.97 \angle -6°$

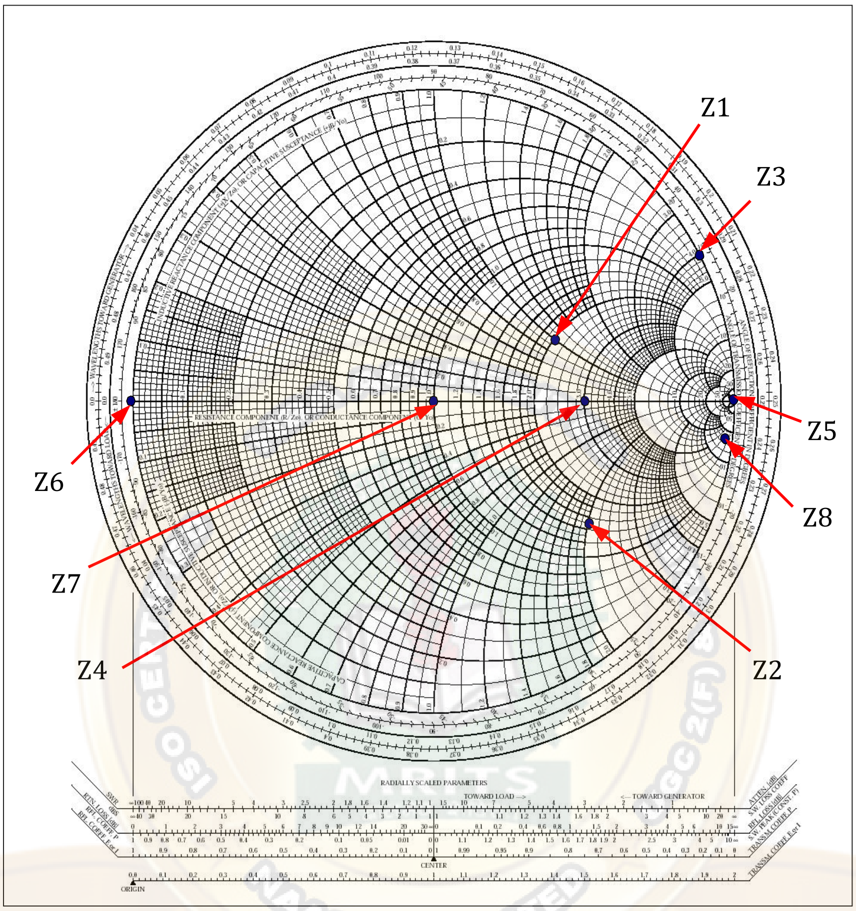


Figure 6: Points plotted on the Smith chart for Example 1.

The Smith chart is constructed by considering impedance (resistance and reactance). It can be used to analyse these parameters in both the series and parallel worlds. Adding elements in a series is straightforward. New elements can be added and their effects determined by simply moving along the circle to their respective values. However, summing elements in parallel is another matter, where admittances should be added.

We know that, by definition, $Y = 1/Z$ and $Z = 1/Y$. The admittance is expressed in mhos or $\wedge^{-1}$

or alternatively in Siemens or S. Also, as Z is complex, Y must also be complex. Therefore

$$Y = G + jB, \tag{10}$$

where G is called the conductance and B the susceptance of the element. When working with admittance, the first thing that we must do is normalize $y = Y/Y_0$. This results in $y = g + jb = 1/z$. So, what happens to the reflection coefficient? We note that

$$\Gamma = \frac{z-1}{z+1} = \frac{(z-1)/z}{(z+1)/z} = \frac{1-y}{1+y} = -\left(\frac{y-1}{1+y}\right). \qquad (11)$$

Thus, for a specific normalized impedance, say $z_1 = 1.7 + j0.6$, we can find the corresponding reflection coefficient as $\Gamma_1 = 0.33 \angle 28^\circ$. From (11), it then follows that the reflection coefficient for a normalized admittance of $y_2 = 1.7 + j0.6$ will be $\Gamma_2 = -\Gamma_1 = 0.33 \angle (28^\circ + 180^\circ)$.

This also implies that for a specific normalized impedance z, we can find $y = 1/z$ by rotating through an angle of 180° around the centre of the Smith chart on a constant radius (see Figure 7).

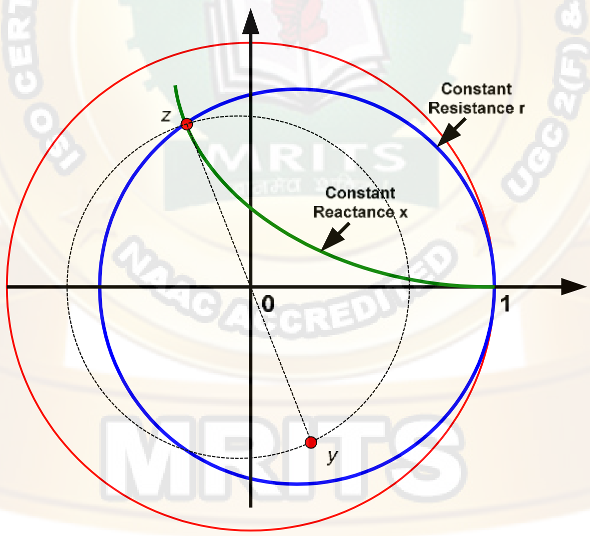


Figure 7: Results of the 180° rotation

Note that while z and $y = 1/z$ represent the same component, the new point has a different position on the Smith chart and a different reflection value. This is due to the fact that the plot for z is an impedance plot, but for y it is an admittance plot. When solving problems where elements in series and in parallel are mixed together, we can use the same Smith chart by simply performing rotations where conversions from z to y or y to z are required.

2 Smith Charts and transmission line circuits

So far we have based the construction of the Smith chart on the definition of the voltage reflection coefficient at the load. The question is: what happens when we connect the load to a length of transmission line as in Figure 8.

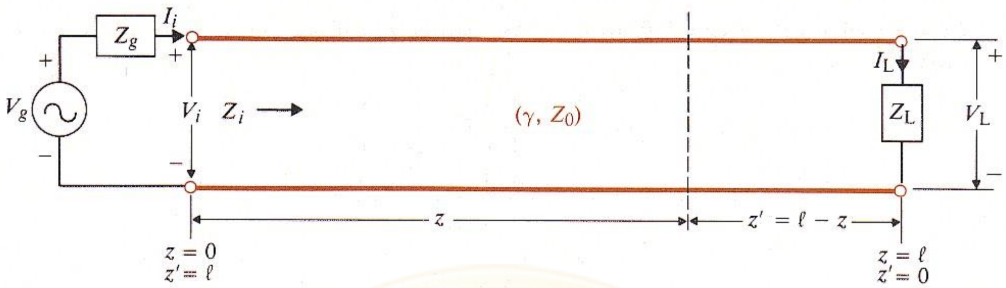


Figure 8: Finite transmission line terminated with load impedance Z_L.

On a lossless transmission line with $k = \beta$, the input impedance at a distance z' from the load is given by

$$Z_i = \frac{V(z')}{I(z')} = Z_0 \frac{1 + \Gamma_L e^{-j2\beta z'}}{1 - \Gamma_L e^{-j2\beta z'}} . \quad (12)$$

The normalised impedance is then

$$z_i = \frac{Z_i(z')}{Z_0} = \frac{1 + \Gamma_L e^{-j2\beta z'}}{1 - \Gamma_L e^{-j2\beta z'}} = \frac{1 + \Gamma_i}{1 - \Gamma_i} . \quad (13)$$

Consequently, the reflection coefficient seen looking into the lossless transmission line of length z' is given by

$$\Gamma_i = \Gamma_L e^{-j2\beta z'} = |\Gamma_L| e^{j\theta} e^{-j2\beta z'} \quad (14)$$

This implies that as we move along the transmission line towards the generator, the magnitude of the reflection coefficient does not change; the angle only changes from a value of θ at the load to a value of $(\theta - 2\beta z')$ at a distance z' from the load. On the Smith chart, we are therefore rotating on a constant $|\Gamma|$ circle. One full rotation around the Smith chart requires that $2\beta z' = 2\pi$, so that $z' = \pi/\beta = \lambda/2$ where λ is the wavelength on the transmission line.

Two additional scales in $\otimes z'/\lambda$ are usually provided along the perimeter of the $|\Gamma| = 1$ circle for easy reading of the phase change $2\beta\otimes z'$ due to a change in line length $\otimes z'$. The scale is marked in "wavelengths towards generator" in the clockwise direction (increasing z') and "wavelengths towards load" in the counter-clockwise direction (decreasing z'). Figure 9 shows a typical commercially available Smith chart.

Each $|\Gamma|$-circle intersects the real axis at two points. Refer to Figure 5. We designate the point on the positive real axis as P_M and on the negative real axis as P_m. Since $x = 0$ along the real axis, both these points represent situations of a purely resistive input impedance, $Z_i = R_i + j0$. Obviously, $R_i > Z_0$ at P_M where $r > 1$, and $R_i < Z_0$ at P_m where $r < 1$. Atthe point P_M we find that $Z_i = R_i = S Z_0$, while $Z_i = R_i = Z_0/S$ at P_m. The point P_M on an impedance chart corresponds to the positions of a voltage maximum (and current minimum) on the transmission line, while P_m represents a voltage minimum (and current maximum). Given an arbitrary normalised impedance z, the value of the r-circle passing through the point P_M is numerically equal to the standing wave ratio. For the example, if $z = 1.7 + j0.6$, wefind that $r = 2$ at P_M , so that $S = r = 2$.

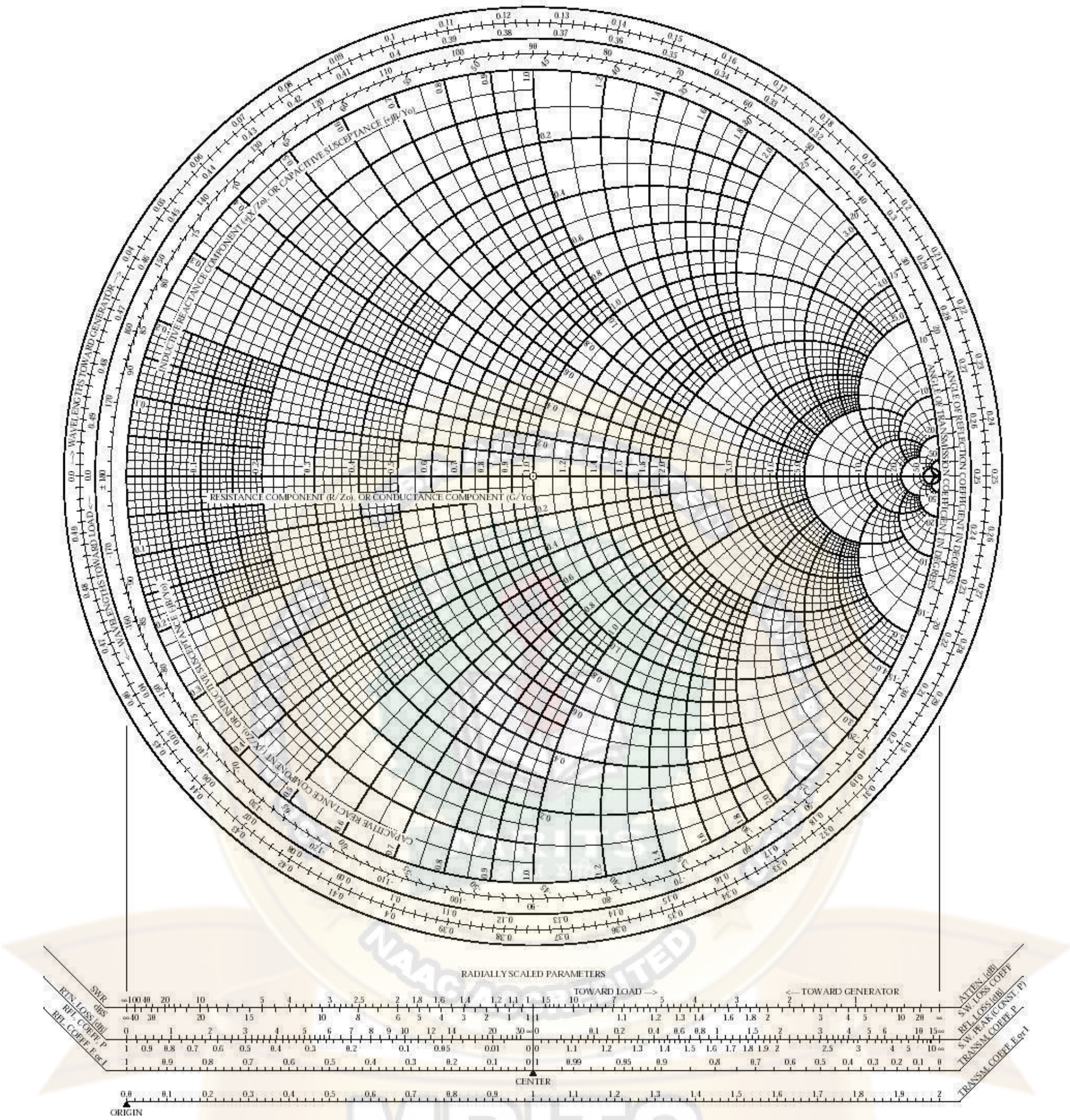


Figure 9: The Smith chart.

Example 2:

Use the Smith chart to find the impedance of a short-circuited section of a lossless 50 ∧ co-axial transmission line that is 100 mm long. The transmission line has a dielectric of relative permittivity $\varepsilon_r = 9$ between the inner and outer conductor, and the frequency under consideration is 100 MHz.

For the transmission line, we find that $\beta = \omega$ = 6.2875 rad/m and

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$\lambda = 2\pi/\beta = 0.9993 \approx 1$ m. The transmission line of length $\sqrt{\mu_0 \varepsilon_0 \varepsilon_r}$ $z' = 100$ mm is therefore

$z'/\lambda = 0.1$ wavelengths long.

- Since $z_L = 0$, enter the Smith chart at a point P_{sc}.
- Move along the perimeter of the chart ($\Gamma = |1|$) by 0.1 "wavelengths towards thegenerator" in a clockwise direction to point P_1.
- At P_1 , read $r = 0$ and $x \approx 0.725$, or $z_i = j0.725$. Then $Z_i = j0.725 \times 50 = j36.3\ \wedge$.

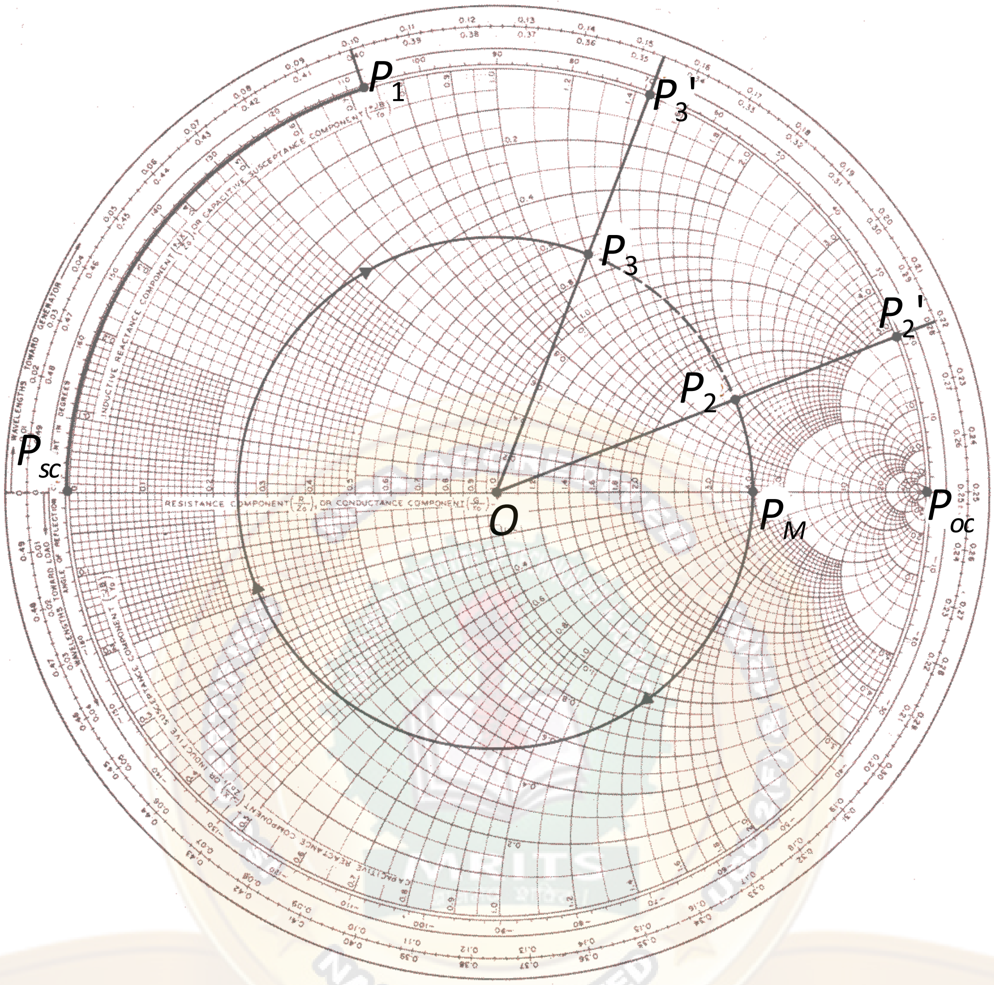

Figure 10: Smith chart calculations for Example 2 and Example 3.

Example 3: A lossless transmission line of length 0.434λ and characteristic impedance 100 ∧ is terminated in an impedance 260 + j180 ∧. Find the voltage reflection coefficient, the standing-wave ratio, the input impedance, and the location of a voltage maximum on the line.

Given $z' = 0.434\lambda$, $Z_0 = 100 \wedge$ and $Z_L = 260 + j180 \wedge$. Then

- Enter the Smith chart at $z_L = Z_L / Z_0 = 2.6 + j1.8$ shown as point P_2 in Figure 10.
- With the centre at the origin, draw a circle of radius $\overline{OP_2} = |\Gamma_L| = 0.6$.
- Draw the straight line OP_2 and extend it to P_2' on the periphery. Read 0.220 on

  "wavelengths towards generator" scale. The phase angle θ of the load reflection may either be read directly from the Smith chart as 21° on the "Angle of Reflection Coefficient" scale. Therefore $\Gamma_L = 0.6\, e^{j21\pi/180} = 0.6\, e^{j0.12\pi}$.

- The $\Gamma = 0.6$ circle intersects the positive real axis OP_{sc} at $r = S = 4$. Therefore the voltage standing-wave ratio is 4.
- The find the input impedance, move P_2' at 0.220 by a total of 0.434 "wavelengths toward the generator" first to 0.500 (same as 0.000) and then further to 0.434–(0.500–0.220)=0.154 to P_3'.

- Join O and P_3 by a straight line which intersects the $|\Gamma| = 0.6$ circle at P_3. Here $r = 0.69$ and $x = 1.2$, or $z_i = 0.69 + j1.2$. Then $Z_i = (0.69 + j1.2) \times 100 = 69 + j120\ \Omega$.
- In going from P_2 to P_3, the $|\Gamma| = 0.6$ circle intersects the positive real axis at P_M where there is a voltage maximum. Thus the voltage maximum appears at 0.250–0.220=0.030 wavelengths from the load.

3 Transmission line impedance matching.

Transmission lines are often used for the transmission of power and information. For RF power transmission, it is highly desirable that as much power as possible is transmitted from the generator to the load and that as little power as possible is lost on the line itself. This will require that the load be matched to the characteristic impedance of the line, so that the standing wave ratio on the line is as close to unity as possible. For information transmission it is essential that the lines be matched, because mismatched loads and junctions will result in echoes that distort the information-carrying signal.

Impedance matching by quarter-wave transformer

For a lossless transmission line of length l, characteristic impedance of $Z_0 = R_0$ and terminated in a load impedance Z_L , the input impedance is given by

$$Z_i = R_0 \frac{Z_L + jR_0 \tan\beta l}{R_0 + jZ_L \tan\beta l} = R_0 \frac{Z_L + jR_0 \tan(2\pi l/\lambda)}{R_0 + jZ_L \tan(2\pi l/\lambda)}. \qquad (15)$$

If the transmission line has a length of $l = \lambda/4$, this reduces to

$$Z_i = R_0 \frac{Z_L + jR_0 \tan(\pi/2)}{R_0 + jZ_L \tan(\pi/2)} = R_0 \frac{Z_L/\tan(\pi/2) + jR_0}{R_0/\tan(\pi/2) + jZ_L} = R_0 \frac{0 + jR_0}{0 + jZ_L} = \frac{(R_0)^2}{Z_L}. \qquad (16)$$

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This presents us with a simple way of matching a resistive load $Z_L = R_L$ to a real-valued

input impedance of $Z_i = R_i$: insert a quarter-wave transformer with characteristic impedance

R_0. From (16), we have $R_i = (R_0)^2 / R_L$, or

$$R_0 = \sqrt{R_i R_L} \quad . \tag{17}$$

Note that the length of the transmission line has to be chosen to be equal to a quarter of a transmission line wavelength at the frequency where matching is desired. This matching method is therefore frequency sensitive, since the transmission line section will no longer be a quarter of a wavelength long at other frequencies. Also note that since the load is usually matched to a purely real impedance $Z_i = R_i$, this method of impedance matching can only be applied to resistive loads $Z_L = R_L$, and is not useful for matching complex load impedances to a lossless (or low-loss) transmission line.

Example 4

A signal generator has an internal impedance of 50 ∧. It needs to feed equal power through alossless 50 ∧ transmission line with a phase velocity of 0.5c to two separate resistive loads o

64 ∧ and 25 ∧ at a frequency of 10 MHz. Quarter-wave transformers are used to match the loads to the 50 ∧ line, as shown in Figure 11.

(a) Determine the required characteristic impedances and physical lengths of the quarter-wavelength lines.
(b) Find the standing-wave ratios on the matching line sections.

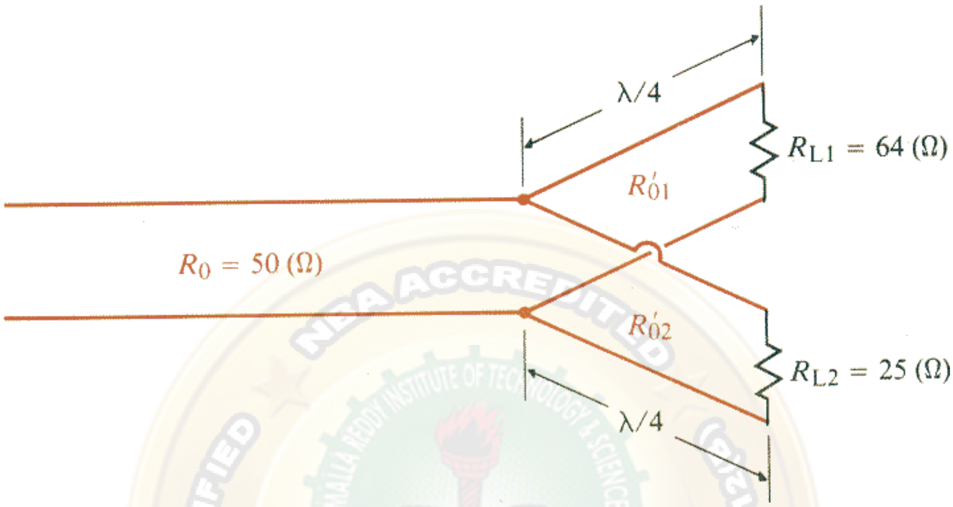

Figure 11: Impedance matching by quarter-wave transformers (Example 4).

(a) To feed equal power to the two loads, the input resistance at the junction with the mainline looking toward each load must be

$R_{i1} = 2R_0 = 100$ ∧ and $R_{i2} = 2R_0 = 100$ ∧

Therefore

$$R'_{01} = \sqrt{R_{i1} R_{L1}} = 80 \wedge$$

$$R'_{02} = \sqrt{R_{i2} R_{L2}} = 50 \wedge$$

Assume that the matching sections use the same dielectric as the main line. We know that

$$u_p = \frac{1}{\sqrt{\mu\varepsilon}} = \frac{1}{\sqrt{\mu_0 \varepsilon_0 \varepsilon_r}} = \frac{c}{2}.$$

We can therefore deduce that it uses a dielectric with a relative permittivity of $\varepsilon_r = 4$.

$$\lambda = \frac{u_p}{f} = \frac{2\pi}{k} = 15 \text{ m}.$$

The length of each transmission line section is therefore $l = \lambda / 4 = 3.75$ m.

(b) Under matched conditions, there are no standing waves on the main transmission line, i.e.

S = 1. The standing wave ratios on the two matching line sections are as follows:

$$\Gamma_{L1} = \frac{R_{L1} - R'_{01}}{R_{L1} + R'_{01}} = \frac{64-80}{64+80} = -0.11$$

$$S_1 = \frac{1+|\Gamma_{L1}|}{1-|\Gamma_{L1}|} = \frac{1+0.11}{1-0.11} = 1.25$$

Matching section No. 2:

$$\Gamma_{L2} = \frac{R_{L2} - R'_{02}}{R_{L2} + R'_{02}} = \frac{25 - 50}{25 + 50} = -0.33$$

$$S_2 = \frac{1 + |\Gamma_{L2}|}{1 - |\Gamma_{L2}|} = \frac{1 + 0.33}{1 - 0.33} = 1.99$$

R16

Code No: 133BJ

JAWAHARLAL NEHRU TECHNOLOGICAL UNIVERSITY HYDERABAD

B.Tech II Year I Semester Examinations, March - 2021NETWORK ANALYSIS

(Electronics and Communication Engineering)

Time: 3 hours Max. Marks: 75

Answer any five questions All questions carry equal marks

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1.a) What is basic cutset matrix? Explain with an example.

b) Draw the T equivalent model for magnetically coupled circuits and explain. [8+7]

2.a) Explain the concept of impedance transformation with an example.

b) In the circuit shown in figure 1, $L_1 = L_2 = 2$ H and M = 0.6 H. Compute i_1 and i_2, if $v_1 = 2\,e^{-2t}, v_2 = 3e^{-3t}$ [8+7]

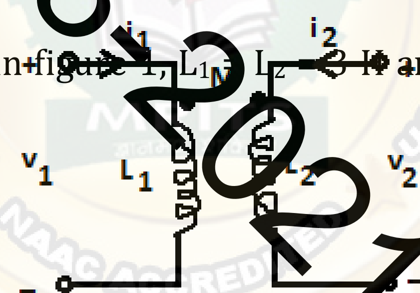


Figure: 1

3.a) Obtain the expression for resonant frequency of RLC series circuit.

b) In the circuit shown in figure 2, find current 'i' at t = 10 sec. [7+8]

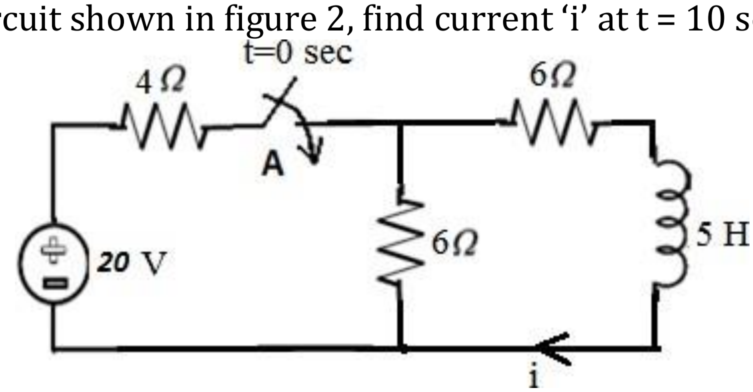


Figure: 2

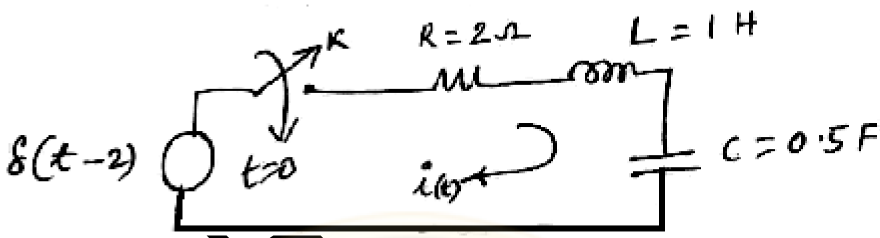
4.a) What is damping factor? Explain the step response of second order system withunderdamped case.

b) Determine quality factor and bandwidth for the parallel RLC resonant circuit. Given

R = 100 Ω, L = 0.2 mH and C = 500 μF. [7+8]

5.a) Define average value of a triangular periodic waveform. Derive the expression for it forsinusoidal waveform.
b) Using Laplace transform techniques, derive the expression for transient current in seriesRL circuit excited by impulse input. [8+7]

6. Obtain the transient response of current for the following network shown below figure 3. [15]



7.a) What is characteristic impedance? Explain its importance in detail .
b) Derive g parameters for the following two port network shown in figure 4. [7+8]

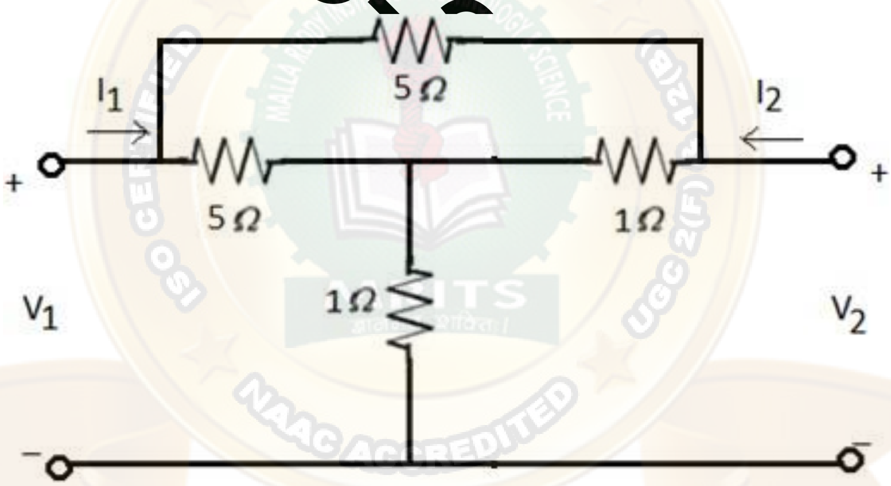


Figure: 4

8.a) Give the classification of attenuators.
b) Discuss in detail about the design of constant HP filter. [6+9]